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A CUSUM scheme for event monitoring

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ABSTRACT

This article presents a single CUSUM scheme (called the *GCUSUM chart*) for simultaneously monitoring the time interval T and magnitude X of an event. For example, a traffic accident may be considered as an event, and the total loss in dollars in each case is the event magnitude. Since the GCUSUM chart is developed based on a synthetic statistic G which is a function of both T and X, this new chart is able to make use of the information about the event frequency, as well as the information about the event magnitude. Moreover, the detection power of the GCUSUM chart can be allocated in an optimal manner between those against T shifts and against X shifts, and between those against small shifts and against large shifts. The performance studies show that the GCUSUM chart is more effective than all other charts in the current literature for detecting the out-of-control status of an event. Furthermore, the GCUSUM chart performs more uniformly for detecting process shifts of different types and sizes. This chart is also easier to be designed and implemented than other CUSUM charts for monitoring both T and X. The GCUSUM chart has the potential to be applied to many different areas, especially to the non-manufacturing and service sectors, such as supply chain management, homeland security, office administration and the health care industry.

1. Introduction

The Statistical Process Control (SPC) chart is an effective on-line monitoring technique widely used in production processes. Recently, SPC chart has found an increasing number of new applications in non-manufacturing sectors (Woodall, 2006; Han et al., 2010; Shu et al., 2010), such as the monitoring of different events (Wu et al., 2010). An event may be a quality problem, an order of products, or a damaging accident of any type. A SPC system or control chart can continuously monitor the collected data of an event, including the time interval T between two consecutive occurrences and the magnitude X of each occurrence. Based on the data analysis, the control chart can decide whether the situation is under control or out of control, or whether any immediate and reinforced action should be taken. Both T and X are random variables. An upward shift (i.e., an increase in frequency (or a decrease in T) and/or an increase in X) indicates a move in a loss direction for a hazardous event. It means a higher than usual rate in damage, cost, or loss incurred by the event. On the contrary, a downward shift (i.e., an increase in T and/or a decrease in X) indicates an improving movement, showing that the severity of the problem is alleviated or improved. In SPC practice, one is usually more interested in detecting an upward shift of an event which has a negative or hazardous effect, because this shift represents a transition to a worsening process status and, as a result, the users should be alarmed as soon as possible. An SPC control chart can assist in decision making to avoid the following two types of errors:

(1) A go-ahead decision for enhancement is made too rashly when the situation is actually still under control. Such a decision is referred to as a *false alarm*, which always leads to unnecessary waste of resources, like money, material and manpower.
(2) A decision for a necessary and immediate action is delayed when the event already becomes out of control (e.g., shift to the loss direction). Such an error may leave the process or system unprotected and lead to serious or even disastrous consequences.

Many SPC control charts can be used to monitor the time interval T of an event, including the exponential chart (Calvin, 1983; Goh, 1987; Chan et al., 2000; Xie et al., 2002), the CRL and RL_2 charts (Bourke, 1991, 2006), the SCRL and Gamma charts (Wu et al., 2001; Zhang et al., 2007), the exponential CUSUM chart (Lucas, 1985; Vardeman and Ray, 1985; Gan, 1992; Borror et al., 2003), the geometric CUSUM chart (Bourke, 2001), the

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exponential EWMA chart (Gan, 1998), and the CQC-r chart (Xie et al., 2002).

As a matter of fact, the effect (e.g., the amount of loss or damage) of an event in a certain time period is, however, not only decided by T (or the frequency of the event), but also by the magnitude X. For example, a fire department must be enhanced and other measures must be in force not only when the fire outbreak becomes very frequent but also when the average damage caused by each outbreak becomes very high. Similarly, the threat of a bird flu becomes critical not only when the outbreak of this infection becomes very frequent but also when the number of poultries infected in each outbreak becomes very high. The control charts that only detect T shifts may not be adequately sensitive as they completely ignore the information about X. The individual X chart used in traditional SPC, as well as the CUSUM, EWMA or SPRT charts, may be adopted to monitor the magnitude X (De Magalhães et al., 2009; Lee et al., 2012; Ou et al., 2012a, 2012b).

Like in a common SPC practice in which the $\overline{X}$&R (or $\overline{X}$&S) combination is used side by side to monitor the mean and variability of a variable, the combined T&X schemes were recently investigated (Liu et al. 2009; Wu et al., 2009a) to simultaneously examine both the time interval and magnitude of an event. The performance studies show that these T&X schemes are considerably more effective and behaves more uniformly in a large shift domain, compared with an individual T or an individual X chart. A single rate chart (Wu et al., 2009b) was also proposed for monitoring the concurrent shifts in both T and X. The rate chart deals with a single statistic R that is the ratio between the sample values of X and T.

While the Shewhart-type control charts are simple in design and implementation, the CUSUM-type charts have been increasingly adopted recently owing to the fact that on-line measurement and distributed computing systems have become the norm in today's SPC applications (Woodall and Montgomery, 1999). Computers remove the difficulties in the designs and implementation of advanced control charts, such as the CUSUM and EWMA charts. Since a CUSUM chart incorporates all the information in the sequence of the sample points by monitoring the cumulative sums of the statistics, it is not only more sensitive to small process shifts than a Shewhart control chart, but also considerably more effective over the whole shift domain. A TC-CUSUM scheme was recently proposed (Wu et al., 2010). It consists of two individual CUSUM chart components, one of which is to monitor the time interval T and another for event magnitude C which is an attribute. This combined CUSUM scheme is much more effective than the Shewhart type combination from an overall viewpoint. A similar TX-CUSUM scheme comprising two separate CUSUM charts (one for monitoring T and another for monitoring X) can be easily designed, in which the event magnitude X is a variable.

Towards this end, this article proposes a single CUSUM chart (the GCUSUM chart) to monitor the concurrent shifts in T and X. The GCUSUM chart is developed based on a single synthetic statistic G that contains the information of both T and X. As demonstrated in this article, this new chart is more effective and robust over the whole shift domain than any other chart for detecting T and/or X, including the TX-CUSUM chart. Moreover, the GCUSUM chart is much easier to be designed and relatively simpler to be operated compared to the TX-CUSUM chart.

The remainder of this article proceeds as follows: the synthetic statistic G is introduced in the next section, followed by the design and implementation of the GCUSUM chart. Then a systematic performance comparison among the GCUSUM chart and several other charts is conducted. Two examples of applications are also illustrated. Discussions and conclusions are drawn in the last section.

2. Introduction to the statistic *G*

Most of the reported works (Xie et al., 2002) assume that T follows an exponential distribution with a single parameter $\lambda > 0$ (i.e., the failure rate or the reciprocal of the mean value of T). An increasing (or decreasing) λ results in a decreasing (or increasing) shift in T. The probability density function f_T and the cumulative distribution function F_T of T are given by

$$f_T(t) = \lambda e^{-\lambda t}, \quad F_T(t) = 1 - e^{-\lambda t}, \quad t \geq 0 \tag{1}$$

On the other hand, the magnitude X is usually assumed to follow a normal distribution with mean μ and standard deviation σ. While the event magnitude X must be positive intrinsically, the normal distribution allows the random variable to be negative theoretically. However, the values of μ and σ of an application will make the probability of $(X < 0)$ almost equal to zero. The probability density function f_X of a normal distribution is calculated by

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-0.5(x-\mu/\sigma)^2}. \tag{2}$$

It is assumed that the time interval T and magnitude X are independent of each other. The in-control values of λ, μ and σ are λ_0, μ_0 and σ_0, respectively. It is also assumed that the variance of X remains unchanged, i.e. $\sigma \equiv \sigma_0$. When an upward shift occurs, the mean μ and failure rate λ will change, i.e.,

$$\mu = \mu_0 + \delta_\mu \sigma_0, \;\; \lambda = \delta_\lambda \lambda_0, \;\; (0 \leq \delta_\mu < \infty, \; 1 \leq \delta_\lambda < \infty) \tag{3}$$

where δ_μ and δ_λ are the shifts in mean and failure rate in terms of σ_0 and λ_0, respectively. When the process is in control, $\delta_\mu = 0$ and $\delta_\lambda = 1$.

Wu and Qu (2010) proposed the following G statistic which is a function of both T and X.

$$G = \eta \left(\frac{1/\lambda_0}{T} \right)^{w_\lambda} + (1-\eta) \left(\frac{X}{\mu_0} \right)^{w_\mu}, \;\; 0 \leq \eta \leq 1, \;\; w_\lambda > 0, \;\; w_\mu > 0 \tag{4}$$

while the term (X/μ_0) is the ratio between a sample value of X and the in-control mean μ_0 (target) of X, the term of $((1/\lambda_0)/T)$ is the ratio between the in-control mean $(1/\lambda_0)$ of T and the sample value of T. Whenever there is an upward shift, that is, a decrease in T with respect to $(1/\lambda_0)$ and/or an increase in X with respect to μ_0, the G value becomes larger. Consequently, both decrease in T (or increase in event frequency) and increase in X can be detected by checking the single statistic G.

The statistic G has three parameters η, w_λ and w_μ. The parameter η is used to allocate the sensitivity of G to T shift and X shift. The exponential w_λ is to adapt the sensitivity of G towards δ_λ of different sizes according to a given shift range of T, and w_μ is to deal with the X shifts. Since T and X follow completely different probability distributions and may have quite different scales in shifts, they should be manipulated separately by w_λ and w_μ. Many researchers (Reynolds and Stoumbos, 2004) found that these exponentials influence the sensitivity of the chart to process shifts of different sizes. They observed that, while $(x-\mu_0)^1$ (or simply $(x-\mu_0)$) is more sensitive to small mean shift, $(x-\mu_0)^2$ is more effective to detect large mean shift. This is because that a larger exponential magnifies a large shift to a more significant degree. Suppose $(x-\mu_0)$ increases from zero to 0.5 (a minor shift), the corresponding change of $(x-\mu_0)^1$ is 0.5, which is larger than the change of (0.25) of $(x-\mu_0)^2$. Conversely, if $(x-\mu_0)$ increases from zero to 4 (a substantial shift), the corresponding change of (4) of $(x-\mu_0)^1$ becomes much smaller than the change of (16) of $(x-\mu_0)^2$. A Shewhart G chart has been developed based on the statistic G (Wu and Qu 2010).

Some other synthetic statistics have also been tried, such as the synthetic statistics without w_λ, or w_μ, or both w_λ and w_μ. However, the numerical tests reveal that the statistic G in Eq. (4) usually