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A lost sales (r, Q) inventory control model for perishables with fixed lifetime and lead time

Chaaben Kouki [a,*], Zied Jemaï [b], Stefan Minner [c]

[a] Montpellier Business School/Montpellier Research in Management, France
[b] Laboratoire Génie Industriel, Ecole Centrale Paris, France
[c] TUM School of Management, Technische Universität München, Germany

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ABSTRACT

We consider a perishable inventory system that operates under stochastic demand, constant lifetime and a constant lead time. The system employs a continuous review (r, Q) inventory control policy where unfilled demands are lost. We investigate the properties of the cost function and present an approximation procedure to find the parameters r and Q that minimize the total cost. We then conduct a numerical analysis to examine the performance of the proposed model and study the sensitivity to changes in the system parameters. We demonstrate the suitability of the proposed approximations compared to optimal (r, Q) parameters obtained by simulation and show that our proposal outperforms another approximation procedure from the literature, in particular for increasing ordering cost and demand variability. The proposed model contributes to the literature by providing a simple and efficient algorithm to compute the best (r, Q) parameters that minimize the total cost. Besides, it can be used in automated store ordering systems.

1. Introduction

Economic sectors such as pharmaceutics, medical goods and consumer goods industries are concerned with the management of perishable products' inventory. In fact, drugs and food, for example, are produced to be consumed within a limited period of time and consequently the impact of perishability on inventory management cannot be disregarded. To illustrate such impact, Roberti (2005) noted that roughly 10% of all perishable goods go to waste before consumers purchase them. In the health care sector, 10.9% of blood platelets processed in the United States outdated without being transfused in 2006 (Fontaine et al., 2009). However, despite the growing literature on perishable inventory control, the fixed life perishability problem remains a complex problem when the product lifetime is longer than two units of time in a periodic review scenario (Nahmias, 1982). Indeed, determining the optimal ordering policy for perishables with deterministic lifetime requires a recursive solution of multi-dimensional dynamic programs. The multi-dimensionality is caused by the need to track the different age categories in stock. Besides, the computation of an optimal policy turns out to be impractical for a real-life situation. The investigations developed so far underline the complexity caused by tracking the different items in inventory by age. We refer to the following literature reviews that give further details pertaining to this difficulty: Goyal and Giri (2001), Karaesmen et al. (2011) and Nahmias (1982).

In this paper, we revisit the fixed life perishability problem and study continuous review (r, Q) inventory systems where products have deterministic lifetimes and excess demands are lost. We derive bounds on the expected number of perished units and expected lost sales and use these bounds to obtain approximations for the expected on-hand inventory level and the expected total cost. Based on the properties of the total cost function, we propose an algorithm to compute the parameters r and Q that minimize the approximated total cost. We show that, compared to similar existing studies, the model we propose performs very well.

Academic literature of inventory control for perishables with deterministic lifetime can be categorized into various classes depending on (i) whether the inventory is reviewed periodically or continuously, (ii) whether replenishment orders arrive instantaneously or after a positive lead time, (iii) the cost components considered, e.g., ordering, inventory holding, outdating and shortage costs. Under periodic review schemes, several heuristics dealing with deterministic lifetime were proposed to avoid the

* Corresponding author at: 2300 Avenue des Moulins, 34080 Montpellier, France. Tel.: +33 467102500; fax: +33 467451356.
E-mail addresses: chaaben.kouki@gmail.com (C. Kouki), zied.jemai@ecp.fr (Z. Jemaï), stefan.minner@tum.de (S. Minner).

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dimensionality of the dynamic problem. For example, Haijema et al. (2007, 2009) used a combination of dynamic programming and simulation to reduce the state space and provided "near optimal" order-up-to-level inventory policies. Haijema (2013) proposed an (r,S) policy for blood platelets where the order quantity is bounded by a minimum and a maximum. Haijema and Minner (2014) compared a variety of hybrid periodic review base stock and constant order policies using simulation based optimization. They proposed two modified based stock policies that achieve a gain of roughly 6% over the base stock policy. Minner and Transchel (2010) investigated a lost sales periodic-review inventory control system with positive lead time and negligible ordering cost. Their model operates both under the FIFO and LIFO issuing policies, i.e. the customers are first served either from the oldest or the youngest items in the inventory. Broekmeulen and van Donselaar (2009) and van Donselaar and Broekmeulen (2012) focused on the periodic review (r, nQ) policy and proposed an approximation for the outdating quantity by combining stochastic modeling, simulation and regression. In Kouki et al. (2014), a periodic review order up to level (T, S) policy is proposed where excess demand is either backordered or lost. Under the assumption of Poisson demand, exponential lifetime and constant lead, the inventory process of this model has Markov properties. Based on this setting, the authors derived the cost components and showed that the consideration of the lifetime variability leads to a significant improvement of the total optimal cost. A detailed literature review for periodic review inventory systems with perishable items is given in Karaesmen et al. (2011) and Nahmias (2011).

Existing studies for inventory control of perishables with continuous review are by Weiss (1980), who investigated the (r, S) policy under zero lead time and Poisson demand. Liu and Lian (1999) extended Weiss's model by considering a general renewal demand process. If the lifetimes and lead time are exponentially distributed, deriving the optimal control parameters becomes somewhat simple because of the application of the Markov renewal theory technique. Several papers appeared under such an assumption (see Kalpakam and Sapna, 1994; Liu and Yang, 1999; Karaesmen et al., 2011).

If a constant and deterministic lead time is introduced, finding an optimal or near optimal policy is analytically complex (both for a random and deterministic lifetime) because of the intractability of the different age categories of items in stock. In fact, there is a limited number of works dealing with continuous review perishable inventory systems with deterministic lead time (Karaesmen et al., 2011). In Table 1, we provide a comparison between the key papers dealing with perishable inventory systems. The initial work by Schmidt and Nahmias (1985) proposed a lost sales $(S-1, S)$ system with Poisson demand and fixed lifetime. Without considering ordering cost, the authors show some properties of the cost function regarding the variation of outdating and shortage cost. This model was extended by Olsson and Tydesj (2010) for backorders. Lian and Liu (2001) considered a general (r, S) perishable inventory model with batch demands. Assuming that the lead time is zero, they constructed a Markov renewal model and proposed a heuristic approach for dealing with the case of constant lead time. Under full backorders, Chiu (1995a) proposed an approximate (r, Q) policy. A similar model has also been studied by Kouki et al. (2013). Under constant lead time, the authors investigated the impact of perishability on the total cost by comparing three (r, Q) models for which they assume infinite, constant and variable lifetime provided by devices called Time Temperature technology (TTIs). Depending on the cost of this technology, the authors showed that considerable gain can be achieved in comparison with (r, Q) models without TTIs. Inventory models with lost sales are discussed in Bijvank and Vis (2011). Particularly for perishable items with deterministic lifetime, Tekin et al. (2001) introduced a (T, r, Q) policy in which a replenishment order of size Q is placed either when the inventory level drops to r, or when T units of time have elapsed since the last instance at which the inventory level hit Q, whichever occurs first. An exact analytical solution of the (T, r, Q) policy is obtained for Poisson demand (slow moving perishable items) and under the assumption that the aging of the batch Q begins after all units of the older batch have been depleted either through demand or by perishing. However, it is too complex to characterize the optimal parameters T, r and Q for general stochastic demand or for perishability starting when orders arrive to the stock.

In the particular case of Poisson demand, Berk and Grler (2008) analyzed the optimal total cost of the (r, Q) policy. They observed that the distribution of the remaining shelf life at epochs when the inventory level is equal to Q has Markovian properties and showed that the remaining shelf life constitutes an embedded Markov process. However, their analysis is only exact when at most one order is outstanding (i.e. $r < Q$). By analyzing this process, they derived a closed form expression for the total cost under the (r, Q) inventory policy with lost sales and positive lead time. Their analysis gives the exact cost expression of the (r, Q) policy under Poisson demand and does not apply for any other types of demand distribution, since then the Markov properties do not hold anymore. Berk and Grler (2008) compared their model to Chiu (1995a) with lost sales and Poisson demand and showed that Chiu's model performs worse if the ordering cost increases. The authors find that Chiu's model deviates from the benchmark (T, r, Q) policy by a maximum of 18% while the optimal (r, Q) policy exhibits a maximum difference of 3.5%. Our paper differs from existing works primarily by considering an (r, Q) inventory system with continuous demand distribution, constant lifetime and constant lead time. Exact analysis of such a system is provided in Berk and Grler (2008) under the assumption of Poisson demand. For other demand distributions, a heuristic solution is proposed in Chiu (1995a). In this paper, we improve Chiu's heuristic by deriving an approximate (r, Q) inventory policy under continuous demand and investigate the sensitivity of the cost function to changes in the system parameters. A similar (r, Q) inventory system has also been studied in Kouki et al. (2013). Their analysis focuses on the backorder case, while we assume that excess demand is lost. Additionally, they implicitly assumed that the inter-arrival time of the demand is discrete and the demand sizes follow a general distribution. In this paper, however, we treated demand as a continuous variable. Finally, our approach to finding the best inventory system parameters is based on determining upper bounds, rather than average values of the expected outdating quantity and expected lost sales.

In the remainder of this paper we formulate our model in Section 2. In Section 3, we investigate some analytical properties of the cost function and provide an approximation procedure for computing the best (r, Q) parameters. We conduct numerical experiments in Section 4 in order to first validate the proposed model, then perform a sensitivity analysis of the cost function, and compare the model we proposed with Chiu's model. In Section 5, we conclude the paper.

2. Model description

We study a single-stage perishable product inventory system. Products have a fixed lifetime of m units of time, which means that they are held in stock during a maximum of m units of time, after which, if they are not consumed, they are disposed of as shown in Fig. 1. The inventory is controlled by an (r, Q) continuous review system: an order of size $Q > 0$ is placed whenever the inventory