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Economic design of Shewhart control charts for monitoring autocorrelated data with *skip* sampling strategies

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ABSTRACT

On-line monitoring of process variability is strategic to achieve high standards of quality and maintain at acceptable levels the number of nonconforming items. Shewhart control charts are the simplest Statistical Process Control (SPC) procedure to achieve this goal. An efficient implementation of a control chart requires the optimal selection of its design parameters. They can be selected according to an economic-statistical objective: an expected total cost per unit of time incurred during production is minimized subject to a statistical constraint limiting the number of false alarms issued by the control chart. This paper investigates the economic-statistical design of Shewhart control charts implementing *skip* sampling strategies for constructing subgroups and monitoring autocorrelated AR(1) processes. Implementing *skip* sampling strategies within a rational subgroup reduces the negative effects of autocorrelation on the statistical performance of the Shewhart control chart. A wide benchmark of examples has been generated as a screening experimental design to study the process and cost factors influencing the selection of the sampling strategy. Regression models have been fitted to the results to help practitioners in the selection of the most convenient sampling strategy. Finally, a sensitivity analysis has been performed to evaluate how the parameters misspecification biases the evaluation of the optimal cost per unit of time.

1. Introduction

Control charts are widely used as online monitoring tools to establish and maintain the statistical control of a process. To run a control chart, its design parameters should be selected. The Shewhart control chart is the most popular *SPC* tool. Its design parameters are the sample size n, the sampling interval h and the width of the control limits k. A control charting procedure consists of plotting a sample statistic obtained from a sample of n measures every h units of time vs. a control interval having width proportional to k. A Shewhart control chart monitoring the sample mean is denoted as the Shewhart $\overline{X}$ chart.

To evaluate the performance of the $\overline{X}$ control chart, Shewhart (1926) made two main assumptions: the observations are independent and the process is subject to assignable causes shifting the process mean off its target. The statistical performance of a control chart measures its speed in the detection of the occurrence of the assignable cause. A well-established measure of statistical performance of a control chart is the Average Run Length (*ARL*) that represents the expected number of samples taken between the occurrence of the special cause and the signal. An efficient control chart should have a small (large) *out-of-control* (*in-control*) *ARL*.

Shewhart did not explore the case when the measures within a sample are autocorrelated. Indeed, in many continuous and batch processes autocorrelation is present and decreases the control chart's statistical performance, see Psarakis and Papaleonida (2007). The effect of the autocorrelation on the chart's performance has been the concern of several researchers. Taking into account the autocorrelation, Vasilopoulos and Stamboulis (1978) and Montgomery and Mastrangelo (1991) suggested to modify the control limits of the Shewhart $\overline{X}$ chart. Bagshaw and Johnson (1975) and Harris and Ross (1991) showed the effect of autocorrelation

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in *CUSUM* schemes. Alwan and Radson (1992) investigated the impact of autocorrelation on the *Type I* error of the $\overline{X}$ chart. English et al. (2000) obtained the *ARLs* of the $\overline{X}$ and *EWMA* charts for autoregressive processes. Apley and Tsung (2002) presented guidelines for the design of the autoregressive T^2 control chart. Noorossana and Vaghefi (2006) studied the performance of *MCUSUM* control charts in presence of autocorrelation. Pacella and Semeraro (2007) used recurrent neural network to detect process mean changes in autocorrelated processes. Soleimani et al. (2009) studied the effect of autocorrelation on the performance of the profile charts when the autocorrelation is overlooked. Costa and Claro (2008) used an *AR*(1) model to describe the observations and obtained the properties of the double sampling $\overline{X}$ chart. Mertens et al. (2009) developed a control chart for monitoring autocorrelated production process data in livestock. Lwin (2011) studied the parameter estimation in *AR*(1) models fitting quality characteristics to be monitored by means of *EWMA* control charts. Costa and Machado (2011) considered an *AR*(1) model to describe the wandering behavior of the process mean and proposed a Markov chain approach to study the performance of a $\overline{X}$ chart with variable parameters or with double sampling monitoring the process. Lin et al. (2011) proposed a model for on-line real-time monitoring in order to recognize unnatural patterns in presence of autocorrelation based on support vector machines. Costa and Castagliola (2011) introduced the idea of modifying the rational subgrouping rules by building samples with non-neighbor items in order to reduce the dependence among observations. Franco et al. (in press) went further in this direction by investigating the statistical performance of a Shewhart $\overline{X}$ control chart implementing a *mixed* sampling strategy, which uses the "1-*skip*" rule as in Costa and Castagliola (2011) and merges the observations from two consecutive samples into one sample having size *n*.

An efficient control chart implementation requires a correct preliminary selection of its design parameters. Duncan (1956) was the first to present a mathematical model for the design of control charts achieving a pure economic objective based on the minimization of a total cost per hour. This approach does not consider the *in-control* statistical performance of the control chart: therefore, the control chart implementation can lead to an excessive number of false alarms with a consequent process variability increase due to unnecessary adjustments, see Woodall (1985). Saniga (1989) and Saniga et al. (1995) introduced the economic-statistical design of a control chart where statistical constraints limiting the expected number of false alarms are imposed to the cost optimization procedure. The economic design of autocorrelated processes has been investigated by several authors: Chou et al. (2001) and Liu et al. (2002) investigated the economic design of $\overline{X}$ control charts for autocorrelated observations, by implementing the Taguchi's quality loss function within the Duncan's economic model; Chen et al. (2007) studied the effect of autocorrelation on the economic design of the $\overline{X}$ control chart with variable sample size and sampling interval (*VSSI*); Torng et al. (2009) economically designed a $\overline{X}$ control chart with double sampling and used the proposed chart to monitor a real correlated process. Recently Lin et al. (2012) studied the economic design of an *ARMA* control chart and Franco et al. (2012) used a genetic algorithm to obtain the economically optimal parameters for Shewhart $\overline{X}$ control charts controlling a wandering process mean based on *AR*(1) model.

This paper investigates the economic performance of the Shewhart $\overline{X}$ control chart implementing the *skip* sampling strategies proposed by Costa and Castagliola (2011) and Franco et al. (in press). The endpoint of this study is to understand which process and cost factors drive the selection of the sampling strategy and to quantify the cost savings associated to each sampling strategy. Furthermore, the influence of the cost and process parameters misspecification is also investigated. The remainder of the paper is organized as follows. Section 2 introduces the First Order Autoregressive *AR*(1) model used to describe the observations; then, it presents the sampling strategies which allow the effect of autocorrelation to be reduced. Section 3 describes the economic model and the exhaustive optimization procedure performed to get the economic-statistical design of the Shewhart $\overline{X}$ control chart. Section 4 shows the numerical analysis and presents a discussion about the results. Section 5 suggests conclusions and future research directions.

2. The Autoregressive model and the skip/mixed strategies

In this paper, we assume that within each sample of size n the observations of the quality characteristic $X \sim N(\mu_0, \sigma_0)$ can be modeled by means of a First Order Autoregressive *AR*(1) model, i.e.

$$X_{i,t} - \mu_0 = \phi(X_{i,t-1} - \mu_0) + \varepsilon_t \quad \text{for } t = 1, 2, \ldots, n \tag{1}$$

where μ_0 is the in-control mean, ε_t, $t = 1, 2, \ldots, n$ are i.i.d. normal $(0, \sigma_\varepsilon)$ random error variables and $\phi \in (0,1)$ is the *known AR*(1) parameter. The sampling interval is selected as long as to assume that $X_{i,t}$ and $X_{i+1,t}$, $(t = 1, 2, \ldots, n)$, are independent, see Alwan and Radson (1992). From the time series theory, the in-control variance σ_0^2 of $X_{i,t}$ is

$$\sigma_0^2 = \frac{\sigma_\varepsilon^2}{1 - \phi^2} \tag{2}$$

According to Alwan and Radson (1992), at the generic ith inspection, the standard-deviation $\sigma(\overline{X}_i)$ of the sample mean $\overline{X}_i = (X_{i,1} + \cdots + X_{i,n})/n$ is given by

$$\sigma(\overline{X}_i) = \frac{\sigma_0}{\sqrt{n} C_2(n, \phi)} \tag{3}$$

where

$$C_2(n, \phi) = \sqrt{\frac{n}{n + \sum_{j=1}^{n-1} 2(n-j)\phi^j}} = \sqrt{\frac{n}{n + 2\left(\left(\phi^{n+1} - n\phi^2 + (n-1)\phi\right) / \left((\phi - 1)^2\right)\right)}} \tag{4}$$

The control limits *LCL* and *UCL* of the two-sided $\overline{X}$ control chart monitoring the autocorrelated *AR*(1) quality characteristic are

$$\begin{aligned} LCL &= \mu_0 - k\frac{\sigma_0}{\sqrt{n}C_2(n,\phi)} \\ UCL &= \mu_0 + k\frac{\sigma_0}{\sqrt{n}C_2(n,\phi)} \end{aligned} \tag{5}$$

where $k > 0$ is the control limit parameter.

After the assignable cause occurrence, the mean is supposed to shift from μ_0 to $\mu_1 = \mu_0 + \delta\sigma_0$ where δ denotes the standardized size of the shift in the mean. According to Costa and Castagliola (2011), the *Type* II error β associated to the $\overline{X}$ chart for autocorrelated data is:

$$\beta = \Phi(k - \delta\sqrt{n}\, C_2(n, \phi)) - \Phi(-k - \delta\sqrt{n}\, C_2(n, \phi)) \tag{6}$$

where $\Phi(\ldots)$ is the cumulative distribution function of the standard normal $N(0,1)$ distribution. The *ARL* of the $\overline{X}$ chart for autocorrelated data is equal to $ARL = 1/P$ where $P = 1 - \beta$ is the power of the chart.

The autocorrelation among consecutive measures within a sample has a negative effect on the chart's sensitivity: Costa and Castagliola (2011) and Franco et al. (in press) proposed to modify the sampling strategy to reduce this sensitivity deterioration. The main idea is to collect one item from the production line and then to skip $s = 0, 1, 2, \ldots$ consecutive items before selecting the next unit belonging to the same sample. Costa and Castagliola (2011)