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A deterministic multi product single machine EPQ model with backordering, scraped products, rework and interruption in manufacturing process

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ABSTRACT

In this paper, an economic production quantity (EPQ) inventory model with interruption in process, scrap and rework is developed and analyzed. The inventory model is for multiple products and all products are manufactured in a unique machine. Obviously, the existence of only one machine results in limited production capacity and shortages. Therefore, shortages are permitted and fully backordered. In this EPQ inventory model, the decision variables are cycle length and backordered quantities of each product and the main objective is to minimize the expected total cost. An easy to use solution procedure is developed for finding the optimal solution. Numerical examples are provided to perform a sensitivity analysis. Finally, some conclusions and future researches are included.

1. Introduction

The first economic production quantity (EPQ) inventory model for a single product-single stage manufacturing system was proposed by Taft (1918). Perhaps Eilon (1957) and Rogers (1958) were the first researchers that studied the multi products-single manufacturing system. Later, this type of the problem was treated extensively in the works of Bomberger (1966), Madigan (1968), Stankard and Gupta (1969), Hodgson (1970) and Baker (1970); just to name a few pioneer works that deal with multi products on a single machine.

Despite its popularity, the EPQ model has limitations and cannot be regarded as a universal inventory model. One limitation is that the production system involved in the classical EPQ inventory model will never manufacture defective products during the production cycle. However, undoubtedly defective products would be made in each production cycle in most real world situations. Therefore, it is worthwhile to study the presence of defective products on inventory models. Due to this fact, there is a growing interest in dealing this issue in EPQ models recently.

According to Haji et al. (2009), it is clear that there are several real world situations in which imperfect quality products should be remanufactured or repaired with an additional cost. In this direction, academicians and researchers have investigated the effects of imperfect quality production, rework and breakdown on EPQ inventory models.

Although, recently there is an enormous emphasis on the implementation of quality control practices in manufacturing systems. We know that till today is difficult to guarantee that one manufacturing system is defect free. Thus, always some products would require a rework process. As discussed earlier, obviously, the presence of defective products is common in many practical manufacturing environments. To address this issue, several researches have focused on the development of EPQ type inventory models involving defective products. For example, Rosenblatt and Lee (1986) present an extended version of the EPQ model. Their inventory model assumes that in some random point in the time the manufacturing process might shift from an in-control to an out-of-control state and then defective products occur. Hayek and Salameh (2001) derive an optimal solution for an EPQ model with rework and imperfect quality products. On the other hand, Jamal et al. (2004) propose an EPQ in which defective products from each production cycle are accumulated until N equal cycles. Then during a rework cycle, defective products are reworked. It is important to point out that all reworked products

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are considered as good as new (serviceable). Hence, they are ready to satisfy the current demand. It should be highlighted that Jamal et al. (2004)'s paper has some errors in the numerical examples. In this direction, Cárdenas-Barrón (2007) corrects the solutions to examples in Jamal et al. (2004). Later, Cárdenas-Barrón (2008) derives in a simple way the Jamal et al. (2004)'s two inventory policies. In a subsequent paper, Cárdenas-Barrón (2009a) develops an EPQ with rework process and planned backorders. It would be useful to note that previous researches have focused on single stage manufacturing systems. Conversely, the multi-stage manufacturing system with rework consideration is also dealt in the inventory literature. See for example the research works of Sarker et al. (2008) and Cárdenas-Barrón (2009b). Several other researches on EPQ inventory model that consider different variants of imperfect production processes are: Gupta and Chakraborty (1984), Chakrabarty and Rao (1988), Cheng (1991), Lee et al. (1997), Salameh and Jaber (2000), Cárdenas-Barrón (2000), Goyal and Cárdenas-Barrón (2002), Sarker et al. (2008), Chung et al. (2009), Liu et al. (2009), Roy et al. (2009), Wang and Tang (2009), Hu et al. (2010), Khan et al. (2011), Sadjadi et al. (2012), Cárdenas-Barrón et al. (2012), Wee et al. (2013), Cárdenas-Barrón et al. (2013a), and Cárdenas-Barrón et al. (2013b); just to name a few recently research works.

Another critical reliability factor is the breakdown of the facilities. This factor may be very disruptive for any automated manufacturing system. In this direction, Chung (1997) presents an EPQ model in which the bounds for production with machine breakdown are studied. Basically, he establishes the optimal upper and lower limits. In another work, Chung (2003) develops a production model in which an approximation to production lot sizing is presented and machine breakdown is one of its main assumptions. Chiu et al. (2007) also develop an EPQ model with random breakdown, rework and scrap production. Other researches about machine breakdown are Bielecki and Kumar (1988), Widmer and Solot (1990), Berg et al. (1994), Chiu et al. (2011), among others.

According to Chiu et al. (2007) in real manufacturing systems the imperfect quality products are manufactured inevitably. Also, they mention that defective products may be reworked or repaired. Hence, the overall production/inventory costs involved can be minimized substantially if rework process is done. In the case of rework there are several works. For example, Chan et al. (2003) present a new EPQ model with pricing, rework and reject situations. Later, Chiu and Chiu (2006) develop an EPQ model with imperfect quality, backordering and failure. In the same year, Islam and Roy (2006) have developed an EPQ model with flexibility and reliability consideration. One year later, Hou (2007) develops an EPQ model with setup cost and process quality as functions of capital expenditure. Liao et al. (2009) investigate an integrated maintenance and production system for the EPQ model with imperfect repair and rework. Chiu et al. (2010) develop a recursive searching algorithm to obtain the optimal run time for an imperfect finite production rate model with scrap, rework, and stochastic machine breakdown. Recently, Chiu et al. (2012) develop a multi-delivery policy into an imperfect EPQ model with partial rework.

In the case of multi product-single machine systems, Haji et al. (2008) study an imperfect manufacturing process with rework in which several products are manufactured on a unique machine. In a subsequent article, Haji et al. (2009) study the optimal batch production with rework subject to a constraint on accumulated defective products. Taleizadeh et al. (2010a) introduce an EPQ model with scrapped products and limited production capacity. At the same time, Taleizadeh et al. (2010b) introduce multi-product single-machine production system with stochastic scrapped production rate, partial backordering and service level constraint. Also in the same year, Taleizadeh et al. (2010c) develop an EPQ inventory model with random defective products, service level constraints and rework process. Recently, Taleizadeh et al. (2012) develop an EPQ inventory model for multi products in a single machine with rework process and Taleizadeh et al. (2013) present an economic production quantity model with repair failure and limited capacity.

In many manufacturing process of the real world, the manufacturing of products with imperfect quality is inevitable. The reworking process and/or scraping of these non-conforming products are considered in different industries. Obviously, setting the appropriate inspection policy on products and/or machine is usually beneficial for reducing the cost imposed by producing defective products. It is well-known that the inspection processes are typically categorized on two types: on line and off line inspection. On line inspection is performed during the production uptime without production interruption. On the other hand, it is important to remark that in many industries it is not possible or it is very expensive to perform the on line inspection. Furthermore, the majority of previous inventory models developed by researchers usually remarked off line inspection as an operation that must be performed at the end of each production runtime in each cycle. Moreover, in many inspection process are accompanied with some predetermined operation like lubrication, machine re-preparation, machine cleaning, preventive maintenance actions and so on which should be performed during the production uptime. Therefore during these operations the production lot is interrupted and will continue after the operation is done. In this study it is considered a new type of off line inspection.

As discussed earlier in the literature review until now no research is done on the jointly considering multi products-single machine with scrap, rework, interruption in process, backlogging situation and off line inspection schedule. Therefore, this paper develops an EPQ inventory model with scrap, reworking of defective products, interruption in process, backorders allowed for a multi products-single machine system. To the best of our knowledge there is not attention to the area of multi products-single machine, specifically on production systems with failure, rework and interruption in process. Thus, this paper intends to serve this purpose.

The remainder of the paper is organized as follows. Section 2 presents the problem definition and notations. Section 3 develops the economic production quantity (EPQ) inventory model with interruption in process, scrap and rework with backordering. Section 4 contains the complete and analytic solution procedure to locate and guarantee the optimal solutions to the EPQ inventory model proposed. In Section 5, two numerical examples are solved and also a sensitivity analysis is conducted. Finally, in Section 6 some conclusions and future researches are given.

2. Problem definition

As it was discussed before, there are many real work situations in which the manufactured imperfect quality products should be reworked or repaired with an additional cost. This paper considers a manufacturing system that generates imperfect products. Furthermore, these defective products are repairable and interruption in manufacturing process will occur. Moreover, it is assumed that there is no interruption during the rework process. Since there is only a single machine the source of regular production and rework, obviously, is the same. During the regular production uptime, a x portion of manufactured products is expected to be defective and is generated randomly at a production rate λ. Among these defective products a θ portion is considered to be scrap and the other portion can