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Joint determination of rotation cycle time and number of shipments for a multi-item EPQ model with random defective rate

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ABSTRACT

This paper addresses the joint determination of a rotation cycle time and number of shipments for a multi-item economic production quantity (EPQ) model with random defective rate. The classic EPQ model considers production planning for a single product with a perfect production process and continuous inventory issuing policy for its finished goods. However, in real vendor–buyer integrated systems, the multi-delivery policy is often used in lieu of the continuous issuing policy, and due to various uncontrollable factors, the generation of defective items is inevitable. Furthermore, in order to maximize machine utilization, management often plans the production of *m* products in turn on a single machine, rather than a single product on one machine, as assumed by the EPQ model. Therefore, by assuming that all defective items produced are scrap, this study aims to jointly determine the optimal rotation cycle time and number of shipments that minimize the long-run average cost for such an imperfect quality multi-item EPQ model. We employ mathematical modeling with the renewal reward theorem and obtain closed-form optimal operating policies for the proposed model. Further, we demonstrate the practical use of the results by using a numerical example and sensitivity analysis.

1. Introduction

The economic production quantity (EPQ) model was introduced (Taft, 1918) to assist the manufacturing firms in deciding the optimal batch size that would minimize the expected production–inventory costs. The classic EPQ model considers production planning for single product with perfect production process, and a continuous inventory issuing policy for its finished goods. However, in real vendor–buyer integrated systems: (1) the multi-delivery policy is often used in lieu of the continuous issuing policy, and (2) due to various uncontrollable factors, generation of defective items is inevitable. Studies that related to different aspects of multi-delivery vendor–buyer systems are surveyed as follows. Goyal (1977) studied an integrated single supplier–single customer problem. He presented a method that is typically applicable to the inventory problems where a product is procured by a single customer from a single supplier using examples to demonstrate his proposed model. Schwarz et al. (1985) studied the system fill-rate of a one-warehouse N-identical retailer distribution system as a function of warehouse and retailer safety stock. They employed an approximation model from a prior study to maximize system fill-rate subject to a constraint on system safety stock. As results, properties of fill-rate policy lines are suggested. They could be used to provide managerial insight into system optimization and as the basis for heuristics. Sarker and Parija (1994) studied a production–shipment system in which raw materials are procured from suppliers and then processed for conversion to finished items. They proposed a model for determining an optimal ordering policy for the procurement of raw materials, as well as for manufacturing a batch size to minimize the total cost for delivering equal shipments of the finished products at fixed intervals to the buyers. Swenseth and Godfrey (2002) showed that straightforward freight rate functions presented in the literature can be incorporated into inventory replenishment decisions without compromising the accuracy of the decision. They also concluded that these functions can be incorporated without adding undue complexity to the decision process. Jaber and Goyal (2008) studied coordination of order quantities amongst the players in a three-level supply chain with a centralized decision process. The first level of the supply chain consists of multiple buyers, the second level of a vendor (e.g., manufacturer), and the third level consists of multiple suppliers. Each supplier supplies one or more items required in the manufacture of the product produced. Their model showed that costs for each level either remain the same as before coordination, or decrease as a result of coordination. Furthermore, they assumed that savings generated from coordination would be distributed among the players of the chain. Additional studies that addressed

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various aspects of supply-chain issues can also refer to the following: Hill, 1995; Viswanathan, 1998; Cetinkaya and Lee, 2000; Bylka, 2003; Sarmah et al., 2006; David and Eben-Chaime, 2008; Sana, 2012; Chiu et al., 2012a; Alghalith, 2013.

Studies that related to production systems with defective items are surveyed as follows. Mak (1985) developed a mathematical model for an inventory system in which the number of units of acceptable quality in a replenishment lot is uncertain and the demand is partially captive. It was assumed that the fraction of the demand during the stock-out period which can be backordered is a random variable whose probability distribution is known. The optimal replenishment policy is synthesized for such a system. A numerical example was used to illustrate the theory. The results indicated that the optimal replenishment policy is sensitive to the nature of the demand during the stock-out period. Hariga and Ben-Daya (1998) studied the EPQ problem in the presence of imperfect processes. The time to shift from the in-control state to the out-of-control state was assumed to be flexible, and they provided distribution-based and distribution-free bounds on the optimal cost. For the exponential case, they compared the optimal solutions to approximate solutions proposed in the literature. Wee et al. (2007) studied an optimal inventory model for application to items with imperfect quality and shortage backordered. An algorithm was developed to derive the optimal replenishment decision for their proposed model. Additional studies that addressed various aspects of imperfect quality and unreliable issues in production systems can refer to the following: Henig and Gerchak, 1990; Chern and Yang, 1999; Chelbi and Rezg, 2006; Chen et al., 2012; Mahata, 2012; Chiu et al., 2012b; Sarkar and Sarkar, 2013.

Further, in order to maximize the machine utilization, management often plans the production of m products in turn on a single machine, rather than production of single product as assumed by EPQ model. Bergstrom and Smith (1970) applied the Linear Decision Rules (LDR) to a multi-item formulation (MDR) which solves directly for the optimum sales, production, and inventory levels for individual items in future periods. They showed that the MDR can seek a solution to maximize profit for the firm over the time horizon by an application in a firm producing a line of electric motors. Gaalman (1978) proposed a model to aggregate multi-item versions of the HMMS (Holt, Modigliani, Muth, and Simon) model. The aggregation technique makes use of the structural properties of the inventory-production part of the model and can be performed regardless of the structure of the work force-total production part. Leachman and Gascon (1988) proposed a heuristic scheduling policy for multi-item, single-machine production systems facing stochastic, time-varying demands. Their dynamic cycle lengths heuristic, integrates feedback control based on the monitoring of inventory levels with the maintenance of economic production cycles. The policy can be applied time period by time period to make decisions concerning which items to produce in what quantities during the next time period. Hennet (2001) formulated a cyclic economic lot-scheduling problem (CELSP) to solve a multistage production planning problem in a job-shop. Under the common cycle approach, each generic job consists of producing an end-product and its part types in the quantity required to meet the demand over the common cycle horizon. The study showed that the CESLP can be solved in a decomposed way. A normalized scheduling problem was solved first and then the optimal value of the period was computed by an explicit formula. The obtained solution can be easily implemented and adjusted to bounded random variations of demands. Studies that related to multi-item production planning and optimization issues have been extensively carried out (see for example, Sambasivan and Schmidt, 2002; Mahapatra and Maiti, 2005; Ertogral, 2008; Jodlbauer and Reitner, 2012). This paper studies a multi-item EPQ model with random defective rate and multi-delivery policy with the objective of joint determination of rotation (common) production cycle time and number of shipments that minimizes the long-run average cost per unit time for the proposed system.

2. Problem statement and modeling

This study examines a multi-item economic production quantity model with random scrap rate and multi-shipment policy. Consider a production process produces m products in turn on a single machine in order to maximize its utilization. All products made are screened and the unit inspection cost is included in the unit production cost C_i. During the manufacturing process of product i (where $i = 1, 2, \ldots m$), an x_i portion of nonconforming items is randomly made at a production rate d_i. All nonconforming items cannot be repaired, they are scrapped at the end of process with a cost C_{Si}. Under ordinary operation, no shortages are allowed, so the constant production rate P_i for product i, must satisfies $(P_i - d_i - \lambda_i) > 0$, where λ_i is annual demand rate for product i, and d_i can be expressed as $d_i = x_i P_i$.

Unlike the classic EPQ model considering a continuous issuing policy to meet the demand, this study adopts a multi-delivery policy. Under our proposed policy, the finished items for each product i can only be delivered to customers if the whole production lot is quality assured at the end of production of each product i. Fixed quantity n installments of the finished batch are delivered at a fixed interval of time during delivery time t_{2i} (refer to Fig. 1). The cost parameters used in this study include: the vendor's unit holding cost h_i for product i, the customer's unit holding cost h_{2i} for product i, production setup cost K_i for product i, unit shipping cost C_{Ti} for product i, and fixed delivery cost K_{1i} per shipment for product i. Additional notation also includes the following:

- T the common production cycle time, one of the decision variables,
- n the number of fixed quantity installments of the finished batch, to be delivered to customers in each cycle, another decision variable,
- Q_i production lot size per cycle for product i,
- H_i maximum level of on-hand inventory in units for product i when regular production process ends,
- t_{1i} the production uptime for product i in the proposed system,
- t_{ni} a fixed interval of time between each installment of finished products delivered during t_{2i}, for product i.
- D_i number of finished items (fixed quantity) for product i distributed to customer per delivery,
- I_i number of left over items for product i per delivery after the depletion during t_{ni},

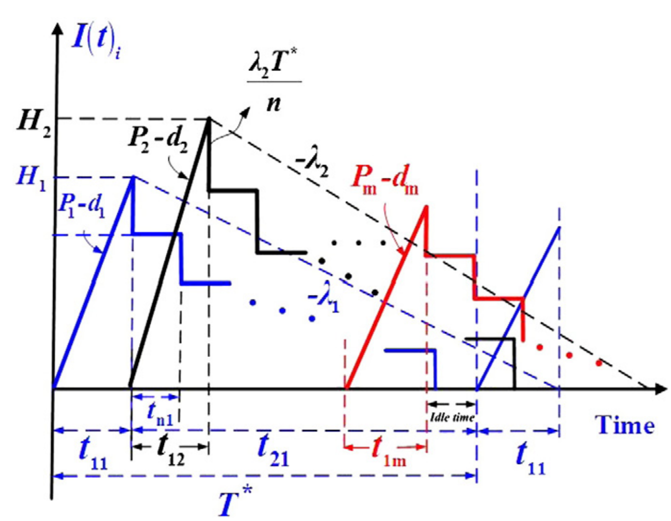

Fig. 1. On-hand inventory of perfect quality items for product i in a common production cycle.