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Effective inventory control policies with a minimum order quantity and batch ordering

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ABSTRACT

In this paper, we consider a single-item periodic-review stochastic inventory system with both minimum order quantity (MOQ) and batch ordering requirements. In each time period, the firm can order either none or at least as much as the MOQ. At the same time, if an order is placed, the order quantity is required to be an integral multiple of a given specific batch size. We first adopt a heuristic policy which is specified by two parameters (s, t). Applying a discrete time Markov chain approach, we compute the system cost and optimize this (s, t) policy under the long-run average cost criterion. We also consider a simpler one-parameter policy, the so-called S policy, which is a special case of the (s, t) policy. In an intensive numerical study, we find that (1) both policies perform well in comparison with other policies and (2) the S policy also performs well and is compatible with the (s, t) policy; only in a few cases where demand variation is small, the latter outperforms the former significantly. We also evaluate the effects of some important parameters on system performance.

1. Introduction

In industries, minimum order quantity (MOQ) and batch ordering, applied independently or simultaneously, are two common requirements made by suppliers, both of which can help companies take advantage of economies of scale and hence reduce costs. The MOQ requirement means that the order quantity must equal or exceed a specified level, if an order is placed. The batch ordering requirement means that the order quantity must be an integral multiple of a specified given batch size.

The application of a MOQ is common in practice. With the prevalence of e-commerce, MOQs are becoming more and more common in our lives, especially in online business-to-business sourcing portals such as alibaba.com, where suppliers often set such requirements. MOQs are also applied in manufacturing industries for products that have short lifetimes or long leadtimes. A well-known example is Sport Obermeyer, a fashion sport ski-wear manufacturer, which has a minimum production level of 600 garments in Hong Kong and 1200 garments in China per order (Zhao and Katehakis, 2006). In fact, MOQ requirements are quite common in China and other low cost manufacturing countries. Low profit margins force manufacturers to pursue large production quantities to break even. On the other hand, batch ordering is also a ubiquitous requirement in industries, because materials often flow in fixed batch sizes in supply chains. For example, raw materials usually arrive at factories in full truckloads, work-in-process is often processed in convenient lot sizes between production stages, and finished goods may be transported in full containers from suppliers to warehouses or distribution centers. Therefore, it is of no surprise that suppliers who apply a MOQ may also require batch ordering. Indeed, our decision to jointly consider both MOQ and batch ordering requirements in this paper is largely motivated by our experience with a wholesale company in Hong Kong. For a variety of products, the firm first replenishes its stock from suppliers and then sells to retail customers, and for most of these products, the firm stipulates both MOQ and batch ordering requirements.

The coexistence of a MOQ and batch ordering has a two-sided effect. On one hand, requiring a MOQ and batch ordering simultaneously helps suppliers reduce the risk of uncertainty and achieve economies of scale. On the other hand, the requirements may have a negative effect on buyers' inventory control, especially when MOQs are relatively large compared with their demand, which is not unusual in practice. Managers in such situations need principles or tools to help control their inventory. However, to the best of our knowledge, no research has investigated inventory systems with both MOQ and batch ordering requirements. Thus, the primary goal of this paper is to fill this gap in the literature. In this paper, we consider a single product stochastic periodic-review inventory system with both MOQ and batch ordering requirements. The selling firm can make a decision at the beginning of

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each time period after reviewing the inventory position. When the firm decides to place an order, the order quantity must satisfy both the MOQ and the batch ordering constraints, where we assume the MOQ is an integral multiple of the batch size. The leftover inventory is carried to the next period and incurs a holding cost, whereas unsatisfied demand is fully backlogged and incurs a backordering cost. The total costs consist of the linear ordering cost, the holding cost, and the backordering cost. The objective is to minimize the long-run average cost of the system.

The optimal policy for the system with only the MOQ requirement, which is partially characterized by Zhao and Katehakis (2006), is rather complicated, even without batch ordering. Therefore, for inventory systems with both MOQ and batch ordering requirements, it is necessary to propose some effective heuristic policies, which is the major contribution of our work. Facing the MOQ requirement, many companies apply the (s,S) type policy to control inventories in practice (Zhou et al., 2007). Based on this, we first propose a two-parameter policy with a similar structure, i.e., the (s,t) policy, where $s<t<s+M$ and M represents the MOQ. The (s,t) policy operates as follows: at the beginning of each period, if the inventory position is less than or equal to s, order a quantity that is just sufficient to bring the inventory position to $s+M$ or above (the inventory position after ordering can be larger than $s+M$, because the order quantity must also satisfy the batch ordering requirement); if the inventory position exceeds s but is no more than t, order exactly M; otherwise, order nothing. We identify the bounds for the optimal t, and propose algorithms to find the optimal values of t and s. We also examine a simpler and more easy-to-use policy, i.e., the S policy, which is a special case of the (s,t) policy. The S policy operates in the same way as the (s,t) policy with $s=S-M$ and $t=S-1$. The numerical study shows that both these polices have close-to-optimal performance in most cases and that there is an overwhelming preponderance to the best (s,S) policy over all examples.

The remainder of this paper is organized as follows. The literature on MOQ and batch ordering is discussed in Section 2. In Section 3, the model description and notations are presented. In Section 4, we propose a two-parameter (s,t) policy and present algorithms to optimize the policy. A simpler one-parameter policy is introduced in Section 5. Numerical examples are conducted in Section 6 to measure the effectiveness of these two policies by comparing them with other policies. Finally, Section 7 concludes the paper by summarizing the findings.

2. Literature review

The existing research on stochastic inventory systems is quite extensive. Here, we mention only a few of the most relevant papers. Many papers focus on problems associated with batch ordering or MOQ separately. The literature related to our paper can be divided into two areas: (1) supply chain inventory management with batch ordering and (2) supply chain inventory management with MOQ.

In the area of batch ordering, Veinott (1965) shows the optimality of the (R,Q) policy for a periodic-review inventory system with batch ordering and no fixed ordering cost. This (R,Q) policy operates as follows: at the beginning of each period, if the inventory position is less than the reorder point R, order the smallest integral multiple of the batch size Q that will bring the inventory position to at least R; otherwise order nothing. Chen (2000) generalizes Veinott's result to multi-echelon systems' settings and demonstrates the optimality of (R,nQ) policies for multi-stage serial and assembly systems where materials flow in fixed batches and the stochastic demands are stationary over time. Chao and Zhou (2009) find the optimal inventory control policy for a multi-echelon serial system with batch ordering and fixed replenishment intervals. They derive a distribution-function solution for its optimal control parameters and design an efficient algorithm for computing those parameters. Huh and Janakiraman (2012) extend the work of Veinott (1965) and Chen (2000) by demonstrating the optimality of echelon (R,nQ) policies for multi-echelon serial systems with nested batch ordering and nonstationary demands. Although it is not optimal in some complex inventory systems with batch ordering, the reorder point, lot-size ordering policy is easy to implement. For this reason, numerous heuristic policies have been proposed, see for example Gallego (1998), Axsäter and Zhang (1999), Shang and Zhou (2010).

In the area of MOQ, Zhao and Katehakis (2006) introduce the concept of M-increasing function and first partially characterize the optimal policy for multiperiod inventory systems with MOQ. For the uncharacterized part, the authors give easily computable upper bounds and asymptotic lower bounds for these intervals. However, for the characterized part, the optimal policy is complexly structured and difficult to implement in practice. Zhou et al. (2009) study a periodic-review inventory system where an additional fixed shipping cost is imposed whenever the order quantity is less than a specified free shipping quantity. The authors characterize the structure properties of the optimal inventory control policy for the single-period model and propose a heuristic policy for multi-period inventory systems. Bradford and Katehakis (2007) study a system where a single supplier has contractual obligations to provide a minimum amount and a maximum amount to all retailers. The authors show that all retailers can be partitioned into three disjoints subsets and provide the optimal allocation for each subset. For other references on MOQ, the reader is referred to Porras and Dekker (2006), Bradford and Katehakis (2006), Okhrin and Richter (2011), Mangione and Penz (2012).

The most closely related papers to our work are Zhou et al. (2007) and Kiesmuller et al. (2011). Zhou et al. (2007) propose a two-parameter heuristic policy for a stochastic inventory system with MOQ requirement and demonstrate that the performance of this policy is close to the optimal policy except for a few cases when the coefficient of the demand distribution is very small. Kiesmuller et al. (2011) propose a simpler policy, which has only one parameter S. This policy works as follows: no order is placed when the inventory position is not less than the level S; otherwise an order is placed to raise the inventory to S. However, if this order is smaller than the MOQ, the order quantity is increased to the MOQ. The authors show the effectiveness of this policy and develop simple newsvendor inequalities for near-optimal policy parameters. However, both Zhou et al. (2007) and Kiesmuller et al. (2011) do not consider batch ordering. To the best of our knowledge, our paper is the first to study stochastic inventory system with both MOQ and batch ordering requirements. To combine the two requirements, we need to tackle the problem of selecting an order quantity that satisfies both the constraints simultaneously. In a system with only the MOQ constraint, the order quantity can be any integer that is larger than or equal to the MOQ. However, with the addition of batch ordering, the firm has to either round up or round down the order quantity to an integral multiple of the given batch size. Therefore, in our model, the order quantities are subject to two kinds of jumps, which makes the analysis much more difficult.

3. Model description

We consider a periodic-review inventory system for a single item with stochastic demand. The demand D in each period is an independent identically distributed (i.i.d.) random variable. The retailer replenishes its stock from a supplier. In our model, when