A production inventory model for deteriorating item with ramp type demand allowing inflation and shortages under fuzziness

Shilpi Pal [a], G.S. Mahapatra [b],*, G.P. Samanta [a]

[a] Department of Mathematics, Indian Institute of Engineering Science and Technology, Shibpur, Howrah 711103, India
[b] Department of Mathematics, National Institute of Technology Puducherry, Karaikal 609 605, India

ARTICLE INFO

ABSTRACT

Economic production quantity (EPQ) is the quantity of a product that should be manufactured in a single batch so as to minimize the total cost. In classical model EPQ only applies where the demand for a product and production rate is constant over the year. But in reality these parameters vary with time in different scenarios. In this paper we have considered a production inventory model for deteriorating items with ramp type demand rate under the effect of inflation and shortages under fuzziness. The deterioration rate is represented by a two-parameter Weibull distribution. As inflation erodes the value of money so we have also considered the effect of inflation when there is shortage in the stock under finite time horizon. Some parameters are vaguely or unclearly defined or whose values are imprecise or determined based on subjective beliefs of individuals. Therefore the inventory model is solved under fuzzy environment to evaluate the optimum solution of the model in different cases. We have optimized our solution by considering production time and production rate as decision variables in two separate cases. While incorporating symmetric triangular fuzzy number we use total λ-integral value to defuzzify the solution. Finally, utility of the model is presented by using some numerical examples and sensitivity analysis and the results are analyzed.

1. Introduction

Now a days, the facts like variation in the demand rate of an item, deterioration, shortages and inflation in the market, are in growing interest. Demand rates are of various types, sometimes it is linear or quadratic time dependent function, or it is a price and stock dependent type of function etc. Recently, many researchers have given considerable attention where the demand rate is a ramp type function of time. This type of demand rate is seen for a new brand item launching in the market. The demand pattern assumed here occurs not only for seasonal product, but also for fashion apparel, computer chips of advanced computer, spare parts, etc. The nature of demand for seasonal and fashionable products is increasing then steady then decreasing and finally becoming asymptotic. The demand of the item increases with time and then stabilizes after some time and ultimately becomes constant. Thus the demand rate is deterministic when any new brand product is launched in the markets, the demand rate linearly depends on time, and later it gets stabilized in the market.

It is noted that the effect of deterioration cannot be ignored in any inventory model. Deterioration means decay, wastage or damage in a way the item cannot be used further for its original purpose. Goods undergo deterioration over time. Food items (like fruits, vegetable, etc),

radioactive substance, drugs, photographic films, etc. undergo spoilage during normal storage. Highly volatile liquids (like alcohol, turpentine, etc) undergo depletion over time by the process of evaporation. Thus decay is a natural phenomenon and it cannot be ignored. But to denote mathematically the leakage failure of the batteries and life expectancy of ethical drugs we use the Weibull distribution. We have considered the model with shortages while the unsatisfied demand is completely backlogged. Due to shortages, partially the inventory is backordered and partially there is lost in sales. Also we have taken into consideration a finite time horizon under inflation. We use inventory carrying cost to determine how much inventory we will keep on hand. Inflation affects the inventory model by increasing carrying costs (because the inflation pushes up the interest rate) which results in a small inventory level. This small quantity is misleading and results in an increase in inventory related costs. Hence, to calculate the optimum order size, cost to carry should be reduced by inflationary impact on interest cost.

In this model we have taken the ramp type function of time as demand rate and deterioration as Weibull's distribution. In reality, the rate and cost fluctuate with time due to various reasons in the market as well as in production unit. So we try to modify the model by fuzzifying the rates and costs to coincide with the realistic situation and check its effect in the model. In the model we have considered the inflationary effect under shortages and thus the lost in sales cannot be ignored. In Section 2 we have taken some assumptions and denoted the variables by some notations that we have used in this paper. In

* Corresponding author. Tel.: +91 9433135327; fax: +91 4368231665.
E-mail address: g_s_mahapatra@yahoo.com (G.S. Mahapatra).

Section 3 we have defined the inventory model with detailed analysis of the model along with the minimization in two different cases. In Section 4 we have observed the effect of fuzziness on the proposed inventory model due to imprecise parameters. It is followed by a numerical example and mathematical analysis has shown graphically the solution of the problem of this paper in Section 4. Lastly in Section 5 we have done the sensitivity analysis of the proposed inventory model.

1.1. Literature survey

In the classical inventory the demand rate was considered as a constant function of time. Covert and Philip (1973), and Philip (1974) considered an inventory model with a constant demand rate. Thereafter researchers started observing various types of demand function. Sometimes demand of the item depends on the on hand stock, price of the items, etc. This type of demand rate was discussed by Hou (2006) considering a stock-dependent demand rate for deteriorating items with shortages. Chung and Wee (2011) scheduled an inventory model for stock dependent selling rate and a replenishment plan for an integrated deteriorating item. Some notable researches were done by Yang et al. (2010) and Wu et al. (2006) in the direction of non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. Sarkar and Sarkar (2013a) have also worked on the model with partial backlogging, time varying deterioration and stock dependent demand. The time varying demand rate actually reflects sales of the item in different phases of the time i.e., demand as well as the sales increases in the growth phase and decreases in the decline phase. This type of demand rate was discussed by many researchers like Hariga (1995) and Bose et al. (1995). Recently researchers are working on the demand pattern of new products launching in the market as a ramp type function of time. Ahmed et al. (2013) proposed a model for new EOQ policy considering ramp type demand rate for deteriorating items with partial backlogging. Skouri et al. (2011a,b) discussed ramp type demand rate with time dependent deterioration. Skouri et al. (2011a,b) also have formulated supply chain models for deteriorating item. Samanta and Bhowmick (2010) formulated a continuous order-level inventory model with ramp type demand rate and deterioration as the Weibull distribution. Agrawal and Banerjee (2011) developed an algorithm considering a two-warehouse inventory model with ramp type demand rate under shortages, where the constant fraction of shortages is backlogged. Wu (2001) worked on an inventory model with ramp type demand rate and Weibull distribution deterioration under shortage and partial backlogging, where it is a variable. An investigation on short life-cycle deteriorating product remanufacturing in a green supply chain inventory control system was developed by Chung and Wee (2011). An EOQ model for deteriorating items with planned backorder level was developed by Widyadana et al. (2011). A production inventory model with random machine breakdown and stochastic repair time was addressed by Widyadana and Wee (2011). Yang et al. (2002) considered the demand of the items as power demand pattern with shortages.

In the beginning researchers did not notice the effect of deterioration in the inventory model. But later they realized that deterioration is a natural phenomenon and so thereafter they started considering the effect of it, in their inventory models. Sarkar and Sarkar (2013b) have worked on variable deterioration and demand. Begum et al. (2012) proposed a replenishment policy with time proportional deterioration and no shortages. An inventory model with a two parameter Weibull distribution was developed by Covert and Philip (1973). It was further developed by Philip (1974). Both of the above papers by Covert and Philip did allow shortages. The effect of deterioration as the Weibull distribution was also dealt by Sharma and Chaudhury (2013) and Skouri et al. (2009). Wu and Ouyang (2000) formulated a replenishment policy for deteriorating items with ramp type demand rate.

Again the classical inventory models did not consider the effect of inflation and time value of the money. But in the last thirty–forty years the economic condition of most of the countries has changed and thus the effect of inflation and time value of money cannot be ignored. In recent past there is a heavy blow to the world economy and thus the time value of money is changing on a day-to-day basis. Buzacott (1975) was the first to consider the inflationary effect in the inventory model assuming constant inflation rate. Datta and Pal (1991); Jolai et al. (2006); Wee et al. (2008); Jaggi et al. (2011); and Neetu and Tomer (2012) have considered an inventory model for deteriorating items under the inflationary effect. Pal et al. (2014b) worked on price and stock dependent demand for deteriorating item under inflation and delay in payment. Yang (2012) worked on two-warehouse partial backlogging inventory models with a three parameter Weibull distribution deterioration under inflation. Uthavakumar and Rameswari (2012) worked on the economic order quantity model for deteriorating items with time discounting. Lin and Lin (2006) studied and proposed a purchasing model, which considers a case of time-varying deterioration, partial back-ordering that depends on the waiting time for backlogging, and time value of money over a finite horizon. Many other researchers have considered the EOQ model under shortage, inflation and finite time horizon. Yang (2011) worked on partial backlogging for a deteriorating item with time varying production rate and demand rate.

Generally, in the inventory model all the cost parameters are taken constant but in reality they are uncertain in nature. So researchers try to introduce the concept of fuzzy in their model. Fuzzy set was first introduced by Zadeh (1965). Then the researchers have used it in various fields. Giannoccaro et al. (2003) considered a fuzzy echelon form in an inventory management system. De and Sana (2013) worked on a fuzzy ordered inventory model with fuzzy shortage and fuzzy promotional index. Mahapatra and Roy (2006) worked on a reliability optimization model using fuzzy multi-objective mathematical programming. Chen and Liu (2007) presented the optimum profit model between the producers and the purchasers for the supply chain system with a pure procurement policy and this paper was further modified by Chen and Lu (2011) where they optimize the profit model by considering production, quantity and sale problem. Mahapatra et al. (2012) developed an imprecise space constraint EPQ model based on an intuitionistic fuzzy optimization technique. Pal et al. (2014a) developed an EPQ model for a ramp type demand with Weibull deterioration under inflation in finite time horizon in crisp and fuzzy environment.

2. Assumptions and notations

We have formulated our model using the following notations and assumptions.

2.1. Notations

D	Demand rate,
K	Production rate,
$\theta(t)$	Distribution of time for deterioration of the item where t denotes time of deterioration,
h	Holding cost per unit per unit time,