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A multiproduct single machine economic production quantity model for an imperfect production system under warehouse construction cost

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ABSTRACT

This paper develops an economic production quantity (EPQ) inventory model for a multiproduct single-machine lot-sizing problem with nonconforming items including scrap and rework, where reworks are classified into several groups based on failure severity. In this inventory model, shortage is allowed and is backordered. The aim is to determine the optimal period length, the lot size, and the allowable shortage of each product so as the total cost, including setup, production, warehouse construction, holding, shortage, reworking, and disposal is minimized. Besides, the available total budget is scarce and that there is a lower bound on the service level of each product. While the convexity is proved, an exact method is proposed to solve the nonlinear programming problem. A numerical example is provided at the end to demonstrate the applicability of the proposed solution procedure as well as to perform a sensitivity analysis.

1. Introduction

Harris (1913) proposed the classical economic order quantity (EOQ) inventory model. It is well-known that this inventory model has assumptions that are not practical in many real-world problems. For instance, it considers that there is no shortage of inventory, the parameters are deterministic, all items are perfect quality. For that reasons, several academicians and researchers have extended it; making it more applicable in real-world inventory problems. Hadley and Whitin (1963) not only provided a summary of the basic EOQ inventory models, but also extended the Harris (1913) inventory model to include shortage. Another extension is due to Abboud and Sfairy (1997) who developed an EOQ inventory model under the effect of time limited free back-orders. They assumed in their research that during a stock-out period the customers would be willing to wait for a limited time to take delivery of their order at no additional charge. In this new variant of the EOQ inventory model, a fraction of the customers are typically lost and hence the lost opportunity costs are considered.

Another unrealistic assumption of the classical EOQ inventory model is that all items are assumed to have a perfect quality. However, deteriorating items are of great importance in the inventory system of modern organizations. Deterioration is regularly defined as defective, scrapped, obsolescence, change, and perishable. Whitin (1953) was the first researcher who introduced deteriorating goods in terms of becoming old-fashioned after a specified period. A decade later, Ghare and Schrader (1963) presented an EOQ inventory model for exponentially decaying inventories with deteriorating items. Covert and Philip (1973) extended the EOQ inventory model for deteriorating items with survival time with a Weibull distribution. Later, Weiss (1982) analyzed an inventory system with deteriorating items. While all parameters are assumed to be deterministic in his work, the maintenance cost is assumed to be a nonlinear function of time. Eroglu and Ozdemir (2007) proposed an EOQ inventory model with backordered shortages and defective items including scrap items with the rate of θ and imperfect items with the rate of $1-\theta$ which are sold at a discounted selling price as a single lot. Interested readers are referred to Goyal and Giri (2001) who presented a review of deteriorating inventory control literature since 1990s, and Bakker et al. (2012) who provided another review on deteriorating inventory

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literature from 2001 to 2012. Recently, Skouri et al. (2014) developed a single-echelon inventory installation under the EOQ strategy with backorders and rejection of defective supply batches to study the effects of supply quality on system cost.

When the items are manufactured in a production system (instead of purchasing) then the so-called economic production quantity (EPQ) inventory model can be used to determine the optimal production quantity. The classical EPQ inventory model was first introduced in 1918 by Taft (1918), in which all the parameters are assumed deterministic and that shortage is not allowed. At the end of the fifties, Rogers (1958) extended the EPQ inventory model to include multiple items. Since then, many extensions of this inventory model have been proposed to relax its constraining assumptions. For instance, Hayek and Salameh (2001) investigated the effect of imperfect quality items on the EPQ inventory model. Their model is derived based on reworking of imperfect quality items produced. Afterwards, Chiu et al. (2007) developed an inventory model that determines the optimal run time in an EPQ model with rework, scrap, and random machine breakdowns.

Li et al. (2008) evaluated the effect of a postponement strategy on a manufacturer in a two-echelon supply chain that utilizes EPQ-based inventory models with planned backorders. After, Cárdenas-Barrón (2009) developed an EPQ inventory model with planned backorders to determine the optimal production quantity in a single-stage manufacturing system. Later, Taleizadeh et al. (2010) developed an EPQ model for a multiproduct single-machine production system with shortage, scrapped production rate, and service level constraint. In their model, shortage is permitted to occur in a mixture of lost sales and backorder. The objective of their research is to determine the permissible shortage level, the period length, and the lot sizes for each product such that the total inventory cost is minimized. Barzoki et al. (2011) presented an EPQ inventory model that considers work in process inventory and imperfect quality products. Sarkar and Moon (2011) introduced an EPQ model with random demand and imperfect quality items under the effect of inflation. Their inventory model assumes that the life cycle of an imperfect item follows a Weibull distribution. Moreover, Uthayakumar and Valliathal (2011) proposed an EPQ inventory model for Weibull deteriorating items in a fuzzy environment. They used triangular fuzzy numbers for costs' coefficients of holding, production, setup, shortage and lost opportunity. Afterward, Widyadana and Wee (2012) developed an EPQ model for deteriorating items with rework process and multiple production setups. They considered the $(m,1)$ policy for one cycle in which the facility can produce items in m production setups and one rework setup.

Tai (2013) proposed two single-period EPQ inventory models for deteriorating/imperfect items with rework. He assumes that imperfect quality items need to be reworked and that the perfect quality items might be deteriorating. The first inventory model is a single production-rework system and the second model is a system that considers several production plants and one rework plant. Besides, Taleizadeh et al. (2013a) developed a single-machine multiproduct EPQ inventory model with imperfect quality items that are reworked. The goal of their research is to minimize the total cost subject to budget and service level constraints. In addition, Taleizadeh et al. (2013b) presented an EPQ inventory model with random defective items, production capacity limitation and failure in repair. Their inventory model obtains the optimal period lengths, order quantities and backordered quantities. Recently, Dey and Giri (2014) developed a single vendor-single buyer EOQ inventory control model with imperfect production process. They assumed the scrapped rate to be an additional control parameter together with the number of shipments from the vendor to the buyer. Wee et al. (2014) proposed an EPQ model with the rework process and non-synchronized screening. They developed a solution procedure to determine the optimal solution. Moreover, Taleizadeh et al. (2014) derived an EPQ model with scrapped items, rework process, interrupted manufacturing process, and backordering with the purpose of minimizing the expected total cost. Other relevant and recent works that address defective items and their rework are Wee et al. (2013), Sarkar et al. (2014), Taleizadeh et al. (2015) and Treviño-Garza et al. (2015).

In an attempt to propose another variant of the EPQ inventory model that is more applicable in real-world inventory problems, a multiproduct single-machine EPQ inventory model for an imperfect production system is proposed in this paper. In this inventory model, each product is assumed in a combination of imperfect and perfect quality items, where imperfect items include scrap and rework. Also, a construction cost is considered to provide a warehouse space for each product. Moreover, there is a limited budget, and that the shortage is allowed in backorder form. The main objective is to determine the optimal period length, the lot size and the allowable shortage for each product so as the total cost, including setup, production, warehouse construction, holding, shortage, reworking, and disposal is minimized. While the convexity is proved, an exact method based on derivatives is proposed to solve the nonlinear optimization problem.

The remainder of this paper is organized as follows. Section 2 defines the problem and the assumptions are explicitly stated too. Section 3 develops the mathematical optimization model. Section 4 presents the solution procedure. Finally, Section 5 solves a numerical example. Finally, Section 6 gives a conclusion and discuses some future research directions.

2. Problem definition and assumptions

The single-machine imperfect production problem of this research assumes that perfect as well as imperfect quality products are produced at certain percentages on a single machine. Furthermore, all imperfect products are classified as reworked and scrapped. In this inventory problem, the annual constant production rate of item i in a regular production time, (P_i), is assumed to be greater than to the annual constant demand rate of product i (D_i), where the annual constant imperfect production rate is $\sigma_i P_i$. Mathematically speaking, $(1-\sigma_i)\ P_i > D_i$ or $a_i = (1-\sigma_i)\ P_i - D_i > 0$. In addition, the σ_i parameter considers two types of parameters, the proportion of re-workable products (α_i^j) and the proportion of scrap items (θ_i). After termination of the regular production, scrapped items are disposed and the rework process starts with the $v_i^1 P_i$, $v_i^2 P_i$, …, and $v_i^m P_i$ rates, where it is assumed no scrapped item is produced during the rework process. As the rework process of a product usually does not require more time compared to its corresponding regular production time, the rework rate is greater than or equal to the regular production rate for all products, i.e. $v_i^j \geq 1$. As a result, the rework production rate $v_i^j P_i$ of the product i is greater than or equal to the demand rate (D_i). In other words, $v_i^j P_i > D_i$ or $y_i^j = v_i^j P_i - D_i > 0$. Additionally, the following conditions are assumed in to model the multiproduct single machine EPQ inventory problem.

1. All products are manufactured on a single machine. Thus, the production cycle length of all items is equal. In other words, $T_1 = T_2 = \ldots = T_n = T$.
2. Shortages are allowed and take the backorder form.
3. The production process includes perfect as well as imperfect quality items.
4. The imperfect quality items include reworked and scrapped items.
5. For each item, there are m types of failures that require rework.
6. The number of re-workable items with percentage rework rate of α_i^1 is less than the quantity of re-workable items with the percentage rework rate of α_i^2 and so on. In other words, $\alpha_i^m \geq \alpha_i^{m-1} \geq \ldots \geq \alpha_i^1$.
7. The rework rates (v_i^j) are proportions of the regular production rate.