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Vertical product differentiation under demand uncertainty☆

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ABSTRACT

In this paper, I examine a quality-then-price game in a fully covered market where firms are uncertain about consumer tastes regarding quality. The equilibrium is characterized under the fixed costs and variable costs of quality improvement, respectively. It is shown that the uncertainty is a differentiation force, and the quality differentiation increases more under variable costs than under fixed costs. In addition, an increase in uncertainty leads to higher profits and higher social welfare regardless of whether under fixed or variable costs. This result contrasts with the lower welfare in the Hotelling model with uncertainty. Finally, an analysis of the case of partial market coverage with uncertainty completes this paper.

1. Introduction

The literature on vertical product differentiation often assumes that firms fully know the consumers' tastes (willingness to pay) for product quality. However, in reality it is too difficult for firms to find out the exact willingness to pay on the part of consumers, even though firms might be able to obtain an estimated distribution of their willingness to pay through market surveys. Accordingly, firms normally face uncertain demand while deciding their product quality.[1]

In this paper, I devote to examining how the uncertainty of firms regarding consumer tastes for quality affects the equilibrium qualities, prices, consumer surplus and social welfare. I introduce the uncertainty of firms over the location of uniformly distributed consumers' tastes into a standard vertically spatial duopoly model.[2] Specifically, the uncertainty over consumer tastes regarding quality is described by a common prior of the lowest quality taste for a uniform distribution of consumers with unit density. It can be viewed as a location uncertainty over the aggregate demand for quality.

I focus on the case in which firms choose qualities simultaneously before observing the exact location of the consumers' preferences and then, after the resolution of the uncertainty, firms compete in prices under complete information. That is, the uncertainty is revealed to both firms after the quality stage but before the price stage. This is because, in reality, it is much easier for firms to change the price than the characteristic of a product. When firms launch new products at wrong prices, the sales data will quickly reflect the mistake, and prices can be readily adjusted up or down in response.

This study aims to draw a comparison with Meagher and Zauner (2004) and to explain the different effects of the demand uncertainty in a fully covered horizontal and vertical model.[3] Accordingly, the major analysis focuses on the case of full market coverage where all consumers are served, as in Champsaur and Rochet (1989, 1990) and Crampes and Hollander (1995), among others. This analysis is applied to the market for necessaries like gas stoves, where consumers are close to being fully served. By contrast, Section 5 briefly examines the case of partial market coverage to explain how the uncertainty may act differently in an uncovered market for products like diamonds or musical instruments.

Different assumptions about the nature of costs have been made in the literature on vertical product differentiation. Some of them, like Shaked and Sutton (1983) and Tseng et al. (2010), assume that the quality improvement is associated with the fixed costs while others, such as Mussa and Rosen (1978) and Lambertini and Orsini (2000, 2001),

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[1] By contrast, Gabszewicz and Grilo (1992), Bester (1998), Cavaliere (2005) and Brouhle and Khanna (2007) consider the consumers' uncertainty over product quality. Bester (1998) shows that unobservable quality reduces the sellers' incentives for horizontal product differentiation, while Gabszewicz and Grilo (1992) characterize the properties of price equilibria by the beliefs of the consumers regarding product quality. Cavaliere (2005) and Brouhle and Khanna (2007) further consider the quality information disparities of consumers.

[2] Carlton and Dana (2008) examine the optimal product line/variety for a monopolist in the presence of sunk unit costs and demand uncertainty, while assuming that the quality tastes of consumers are identical. They show that demand uncertainty and the sunk costs of production induce product variety.

[3] Meagher and Zauner (2004) examine the effects of the demand uncertainty in a Hotelling model with full market coverage. This paper generalizes their analysis to a vertically spatial environment.

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specify that the quality improvement is associated with the variable costs. Moreover, the linear and quadratic forms of costs are often assumed in the literature.

In this paper, I first characterize the equilibrium of a quality-then-price game under uncertainty for both quadratic fixed and variable costs in a fully covered market. The results show that the quality differentiation increases with the demand uncertainty, and it increases more under variable costs than under fixed costs. Moreover, the uncertainty leads to higher profits and welfare regardless of whether under fixed or variable costs. However, it has different effects on the quality, price and consumer surplus of the low-quality product in these two cases. In addition, by comparing the results of the variable-cost case with Meagher and Zauner (2004), the welfare increases with uncertainty in a vertical model, which contrasts with the declining welfare in a horizontal model. This is because, when the uncertainty increases, there is an additional positive effect on the actual demand/consumption for quality in a vertical model. Furthermore, turning to the case with partial market coverage, the quality differentiation also increases with uncertainty regardless of whether under fixed or variable costs. However, in contrast to the fully covered case, the qualities of the two products (the high- and low-quality ones) both increase in an uncovered market due to the competition from an outside good with zero quality and price. In addition, it is found that the uncertainty has no effect on the equilibrium when there is a linear rather than a quadratic form of costs.

The rest of this paper is organized as follows. Section 2 outlines the framework of our model. Section 3 characterizes the subgame perfect Nash equilibrium (SPNE) under uncertainty for the fixed cost of quality improvement. Section 4 characterizes the SPNE under uncertainty for the variable costs of quality improvement, and draws a comparison with Meagher and Zauner (2004) to explain how the demand uncertainty acts differently in fully covered horizontal and vertical models. Section 5 provides an examination to discuss the case of partial market coverage with uncertainty. Section 6 concludes.

2. Model

There are two firms, H and L, producing quality-differentiated goods, each with a quality denoted by q_i, $i = H,L$, and $q_H > q_L$. A continuum of consumers indexed by θ is uniformly distributed in the interval $[\underline{\theta}, \underline{\theta}+1]$ with unit density, where $\underline{\theta}>0$. The parameter θ represents the consumers' willingness to pay for quality. Consumer θ has unit demand for products and either buys a unit from firm H or buys one from firm L; his utility is given by $\theta q_i - p_i$ when he purchases a unit of quality q_i at price p_i.

The two firms play a two-stage game where they simultaneously choose their product qualities in the first stage, and then compete in prices. There is a marginal consumer indexed by $\hat{\theta} = \frac{p_H - p_L}{q_H - q_L}$ where the consumer is indifferent between buying from firm H and buying from firm L. The consumers with $\theta \geq \hat{\theta}$ will purchase the high-quality good, and those with $\theta < \hat{\theta}$ will buy the low-quality good.

In addition, there is an uncertainty regarding consumers' preferences. The uncertainty is over the location of the uniformly distributed tastes on the quality of the consumers, which implies that firms are uncertain about the aggregate demand of the uniformly distributed consumers. Specifically, I treat the lowest willingness to pay of consumers, $\underline{\theta}$, as a random variable, with support on $[\underline{D}, \overline{D}]$, contained in [0, 1/2] of the continuous density function $f(\underline{\theta})$ with $E(\underline{\theta}) = \mu$, and $Variance(\underline{\theta}) = \sigma^2$. Similar to Meagher and Zauner (2003), it can be shown that the size of the support of the uncertainty distribution implies that $\sigma^2 \leq \frac{1}{4}$.[4]

As already mentioned, I consider that the uncertainty is revealed to both firms after the location but before the price game, since the product prices can be flexibly adjusted in reality.

[4] The uncertainty over the location of the distribution of consumers is (relatively) small to avoid distributional assumptions regarding the density function.

3. Fixed costs of quality improvement

In this section, I first assume that the cost of quality improvement falls upon the fixed costs, and specify a quadratic form, αq_i^2, for tractability as in Tseng et al. (2010), among others. Moreover, each firm chooses its quality of product from a wide interval $q_i \in [\overline{q}, \underline{q}]$ where $\overline{q}$ and $\underline{q}$ denote the lower and upper bounds of the quality levels, respectively.[5] Thus, I may write down the profits of firms H and L as follows:

$$\pi_H = p_H\left(\underline{\theta} + 1 - \hat{\theta}\right) - \alpha q_H^2 \tag{1}$$

$$\pi_L = p_L\left(\hat{\theta} - \underline{\theta}\right) - \alpha q_L^2 \tag{2}$$

Using backward induction, I first solve the prices in the second stage:

$$p_H = \frac{1}{3}(q_H - q_L)(2 + \underline{\theta}) \tag{3}$$

$$p_L = \frac{1}{3}(q_H - q_L)(1 - \underline{\theta}) \tag{4}$$

Accordingly, the marginal consumer is

$$\hat{\theta} = \frac{1}{3}[1 + 2\underline{\theta}] \tag{5}$$

Substituting Eqs. (3), (4) and (5) into Eqs. (1) and (2), I obtain

$$\pi_H = \frac{1}{9}(q_H - q_L)(2 + \underline{\theta})^2 - \alpha q_H^2$$
$$\pi_L = \frac{1}{9}(q_H - q_L)(\underline{\theta} - 1)^2 - \alpha q_L^2$$

At the quality stage, the two firms make their decisions under the demand uncertainty. The expected profits of firms H and L are, respectively,

$$E\pi_H = \int_{\underline{D}}^{\overline{D}} \left\{ \frac{1}{9}(q_H - q_L)(2 + \underline{\theta})^2 - \alpha q_H^2 \right\} f(\underline{\theta}) d\underline{\theta}$$
$$= \frac{1}{9}(q_H - q_L)\left(4 + 4\mu + \mu^2 + \sigma^2\right) - \alpha q_H^2$$
$$E\pi_L = \int_{\underline{D}}^{\overline{D}} \left\{ \frac{1}{9}(q_H - q_L)(\underline{\theta} - 1)^2 - \alpha q_L^2 \right\} f(\underline{\theta}) d\underline{\theta}$$
$$= \frac{1}{9}(q_H - q_L)\left(1 - 2\mu + \mu^2 + \sigma^2\right) - \alpha q_L^2$$

Then, each firm chooses a quality to maximize its expected profit. By solving the first-order conditions, I establish the equilibrium below:

Proposition 1. *The equilibrium qualities of firms H and L are, respectively,*[6]

$$\left(q_H^f, q_L^f\right) = \left(\frac{\sigma^2 + (2 + \mu)^2}{18\alpha}, \underline{q}\right). \tag{6}$$

Accordingly, the expected prices are

$$\left(Ep_H^f, Ep_L^f\right) = \left(\frac{(2 + \mu)\left[\sigma^2 + (2 + \mu)^2 - 18\alpha\underline{q}\right]}{54\alpha}, \frac{(1 - \mu)\left[\sigma^2 + (2 + \mu)^2 - 18\alpha\underline{q}\right]}{54\alpha}\right),$$

[5] Note that the quality interval $[\overline{q}, \underline{q}]$ is assumed to be wide enough to let us focus on the interior solution of firms' quality choices whenever there is one.

[6] The second-order conditions are satisfied.