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Economic production quantity models for deteriorating items with up-stream full trade credit and down-stream partial trade credit

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ABSTRACT

In practice, in order to reduce default risks with credit-risk customers, a seller (e.g., a manufacturer or a retailer) frequently requests its credit-risk customers to pay a fraction of the purchase amount at the time of placing an order as collateral deposit, and then grants a permissible delay on the outstanding balance (i.e., a down-stream partial trade credit). By contrast, the seller usually receives a permissible delay on the entire purchase amount from the supplier (i.e., an up-stream full trade credit). In this paper, we propose an economic production quantity (EPQ) model for deteriorating items in a supply chain with both up-stream and down-stream trade credit financing. By using fractional programming results, we can prove that the optimal solution not only exists but also is unique. Moreover, we propose three discrimination terms to identify the optimal solution among possible alternatives. Finally, some numerical examples are presented to highlight the theoretical results and managerial insights.

1. Introduction

Financial supply chain management and working capital management are increasingly recognized as important means to increase profitability in a supply chain. The physical product flow has long been addressed by researchers and practitioners. However, now companies have identified the financial side of the supply chain as a promising area for improvements. By actively managing payment terms and working capital requirements, managers can influence financial performance and achieve significant cost savings. The permissible delay in payments (i.e., trade credit) allows a buyer to accumulate revenue and earn interest during the credit period. However, beyond this credit period the seller charges the buyer interest on the unpaid balance. Hence, from the buyer's perspective, a permissible delay in payments reduces its holding cost, and thus is a powerful promotional tool to attract new customers, who consider it as an alternative incentive policy to price discounts. On the other hand, from the seller's perspective, although offering trade credit increases its opportunity cost due to interest loss during the credit period, it reduces its buyer's holding cost, attracts new customers, and in turn increases its profit.

In 1913, the economic order quantity (EOQ) was first proposed by Harris (1913). Since then prolific extensions of his EOQ model have been developed by researchers. Grubbstrom (1980) built an inventory model with two trade credit periods with no optimization. Goyal (1985) obtained the retailer's optimal order quantity in an EOQ model when the supplier offers a permissible delay in payments. Aggarwal and Jaggi (1995) extended the EOQ model with trade credit financing from non-deteriorating items to deteriorating items. Jamal et al. (1997) further generalized the EOQ model to allow for shortages. Chang et al. (2003) developed an EOQ model for deteriorating items under supplier credits linked to ordering quantity. Huang (2003) proposed an inventory model by assuming that the supplier offers the retailer a permissible delay and the retailer also provides its customers another permissible delay to stimulate demand. Ouyang et al. (2006) established an optimal ordering policy for deteriorating items under trade credits. Liao (2007) presented an economic production quantity (EPQ) model for deteriorating items under permissible delay in payments. Teng et al. (2009) proposed an EOQ model with two warehouses and solved the problem by an arithmetic–geometric inequality method. Hu and Liu (2010) presented an EPQ model with permissible delay in payments and allowable shortages. Teng et al. (2011) extended an EOQ model for stock-dependent demand to supplier's trade credit with a progressive payment scheme. Skouri et al. (2011) studied supply chain models for deteriorating items with ramp-type demand rate under permissible delay in payments. Teng et al. (2012a) discussed vendor-buyer inventory models with trade credit financing under a non-cooperative and an integrated environments. Concurrently, Teng et al. (2012b) established an EOQ model with trade credit financing for increasing demand. Min et al. (2012) developed an EPQ model with

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inventory-level-dependent demand and permissible delay in payments. Tsao (2012) considered manufacturer's production and warranty decisions for an imperfect production system under system maintenance and trade credit. Mahata (2012) proposed an EPQ model for deteriorating items by assuming that the retailer obtains a full trade credit offered by the supplier and offers a partial trade credit to his/her customers. Recently, Chung and Cárdenas-Barrón (2013) presented a simplified solution procedure to an EOQ model for deteriorating items by Min et al. (2010) with stock-dependent demand and two-level trade credit. Chern et al. (2013) established Stackelberg solution in a vendor-buyer supply chain model with permissible delay in payments. Ouyang and Chang (2013) proposed an optimal production lot with imperfect production process under permissible delay in payments and complete backlogging. Chen et al. (2013a) established the retailer's optimal EOQ when the supplier offers conditionally permissible delay in payments link to order quantity. Concurrently, Chen et al. (2013b) attempted to overcome some shortcomings of mathematical model and expressions in Liao et al. (2012).

In this paper we propose an EPQ model for deteriorating items in a supply chain in which a retailer receives a full trade credit from its supplier and simultaneously offers a partial trade credit to his/her customers. This model is closely related to that of Mahata (2012) but the interest earned and interest payable have been calculated in an different and, according to our opinion, more properly way. By applying convex fractional programming results, we obtain the necessary and sufficient conditions of an optimal solution and propose three discrimination terms to identify the global minimum solution among different alternatives. Finally, some numerical examples are used to illustrate the theoretical results and managerial insights.

2. Notation and assumptions

For simplicity, the notation and the assumptions used through the paper are presented below.Notation

D	demand rate in units per year
P	production rate in units per year, $P>D$
ρ	fraction of non-production time $=1-D/P$
A	set-up or ordering cost in dollars per order
h	holding cost in dollars per unit per year excluding interest charges
c	purchase cost per unit in dollars
p	selling price per unit in dollars, $p>c$
M	up-stream trade credit in years offered by the supplier to the retailer
N	down-stream trade credit in years offered by the retailer to its buyers
I_c	interest rate charged per dollar per year
I_e	interest rate earned per dollar per year
α	fraction of total purchase cost which the buyer must pay at the time of placing an order, $0 \le \alpha \le 1$
$1-\alpha$	fraction of total purchase cost which the buyer has a permissible delay of N years
t_1	time in years at which production stops
θ	constant deterioration rate, $0 \le \theta < 1$
T	replenishment cycle time in years
T^*	optimal replenishment cycle time
$TRC(T)$	seller's annual total relevant cost in dollars
$TRC^*(T^*)$	seller's optimal annual total relevant cost in dollars

2.1. Assumptions

(1) The demand rate is known and constant.
(2) Shortages are not allowed.
(3) Time horizon is infinite, and replenishments are instantaneous.
(4) A bank in general loans money only on the retailer's expected receivable revenue (i.e., the revenue received from future sales, which is not including deteriorated items). Therefore, the retailer's interest charged is based on the non-deteriorated items, which is not the entire on-hand inventory as that includes deteriorated items.
(5) The seller receives a full credit period of M years from its supplier, and in turn provides a partial trade credit to its credit-risk customers who must pay α portion of the total purchase cost at the time of placing an order as collateral deposit, and then receive a permissible delay of N years on the outstanding amount. Notice that to good-credit customers, the seller may provide a full trade credit in which we simply set $\alpha=0$. Hence, our proposed model includes the special case in which the seller offers a down-stream full trade credit to its customers.
(6) If $M \ge N$ then the seller deposits the sales revenue into an interest bearing account. If $M \ge T+N$ (i.e., the permissible delay period is longer than the time at which the retailer receives the last payment from its customers), then the seller receives all revenue and pays off the entire purchase cost at the end of the permissible delay M. Otherwise (if $M \le T+N$), the seller pays the supplier the sum of all units sold by $M-N$ and the collateral deposit received from N to M, keeps the profit for the use of the other activities, and starts paying for the interest charges on the items sold after $M-N$.
(7) If $N \ge M$, then the seller finances and pays its supplier the entire amount of the delayed payment $(1-\alpha)cDT$ at the end of the trade credit M, and then pays down the loan after time N at which the seller starts to receive sales revenue from its customers. For the collateral deposit, the seller deposits the sales revenue into an interest bearing account until the end of the permissible delay M. If $T \ge M$, then the seller pays the supplier all units sold by M, keeps the profit for the use of the other activities, and starts paying for the interest charges on the items sold after M.

3. Mathematical formulation of the model

During the production period $[0,\ t_1]$, the inventory level is affected by production, demand, and deterioration. The evolution of the inventory level can be described by the following differential equation:

$$\frac{dI(t)}{dt}+\theta I(t)=P-D,\ 0 \le t \le t_1, \tag{1}$$

with the initial inventory level $I(0)=0$.

Next, during non-production period $[t_1,T]$, the inventory depletes by the combined effect of demand and deterioration. Consequently, the change in the inventory level is described by the following differential equation:

$$\frac{dI(t)}{dt}+\theta I(t)=-D,\ t_1 \le t \le T, \tag{2}$$

with the ending inventory level $I(T)=0$.

The solutions of the above differential equations are respectively:

$$I(t)=\frac{P-D}{\theta}(1-e^{-\theta t}),\ 0 \le t \le t_1, \tag{3}$$

and

$$I(t)=\frac{D}{\theta}(e^{\theta(T-t)}-1),\ t_1 \le t \le T. \tag{4}$$

From the continuity of the inventory level at time t_1 the following relation between t_1 and T prevails:

$$t_1=\frac{1}{\theta}\ln\left[1+\frac{D}{P}(e^{\theta T}-1)\right] \tag{5}$$

For the derivation of the seller's annual total relevant cost, the mathematical expressions of set-up cost, holding cost (excluding