Int. J. Production Economics 155 (2014) 172–184

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

An Economic Order Quantity model with partial backordering and all-units discount

Ata Allah Taleizadeh [a,*], David W. Pentico [b]

[a] School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran
[b] Palumbo-Donahue Schools of Business, Duquesne University, Pittsburgh, PA 15282, USA

ARTICLE INFO

Article history:
Received 7 May 2013
Accepted 16 January 2014
Available online 27 January 2014

Keywords:
Inventory
Pricing
EOQ
Full backordering
Partial backordering
All-units discount
Price

ABSTRACT

The classic Economic Order Quantity model assumes that the unit purchasing cost is not based on the order quantity. In practice, a supplier may offer purchasers an all-units discount. We develop a model and solution procedure for the EOQ with all-unit discounts and partial backordering at a constant rate. We show, and illustrate with a numerical example, how that model can be used to find the solution for the EOQ with discount and full backordering. A multi-factor experiment is used to gain insights into the improvement in performance of the proposed procedure relative to using the basic EOQ or the EOQ with full backordering and to identify the model parameters that have the greatest impact on the model's performance.

1. Introduction and literature review

Two topics that have separately received considerable attention in the inventory control literature are quantity discounts and partial backordering of shortages. Here we develop a model that combines these two extensions of the basic EOQ model developed by Harris (1913).

A supplier may offer a buyer a discounted price if a large-enough quantity of an item is purchased. Several types of single-item quantity discount approaches are used in practice and have been discussed in the literature, among which are all-units discounts, incremental discounts, and standard-quantity discounts. If the same discounted price applies to all the units purchased, it is known as an all-units discount and only the price offered for the specific range within which the order quantity falls is applicable for finding the order's total cost. If a discount applies only to units in excess of a specified amount, it is known as an incremental discount and the prices offered in all the different order-quantity ranges up through the range that includes the actual order quantity are applicable in finding the order's total cost. With standard-quantity discounts an order may be a combination of several different standard quantities, such as units, boxes, and cases, each of which has a specific price, with the cost per unit being lower for larger standard quantities. The total order cost will be determined by how many of each standard quantity there are in the order.

Benton and Park (1996) surveyed papers published through 1993 on lot sizing problems in which at least one type of quantity discount (incremental and all-units policies) is used. More recent research includes the following. Weng (1995) developed models to determine the optimal all-units and incremental discount policies and investigated the effects of those policies on price sensitive demands. Tersine et al. (1995) developed an EOQ model with fully backordered shortages when all-units, incremental, and freight discounts are offered by the seller. Chung et al. (1996) developed models for the classic dynamic lot sizing problem under all-units and incremental discount policies. Lin and Kroll (1997) developed four different mathematical models for the classic newsboy inventory problem with all-units and incremental discounts when the penalty cost for each unit of shortage is zero or positive. Burwell et al. (1997) developed a lot-size model with price sensitive demand under all-units and freight discount. Chang (2013) revisited the research in Burwell et al. (1997) and (1) provided counterexamples to show that adopting their algorithm to determine overall optimal lot size and selling price may not achieve the goal of maximizing profit, and (2) proposed a new algorithm to determine an exact solution. Chan et al. (2002) investigated an economic lot-sizing problem with a special class of piecewise linear ordering costs, which they referred to as the class of modified all-unit discount cost functions. Wang and Wang (2005) studied a supplier's optimal quantity discount policies, both all-units and incremental, for a group of independent and heterogeneous retailers when each retailer faces a demand that is a decreasing function of its retail price. Burnetas et al. (2007) studied how a supplier can use a quantity discount schedule, both

* Corresponding author.
E-mail addresses: taleizadeh@ut.ac.ir (A.A. Taleizadeh), pentico@duq.edu (D.W. Pentico).

0925-5273/$ - see front matter © 2014 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.ijpe.2014.01.012

all-units and incremental, to influence the stocking decisions of a downstream buyer who faces a single period of stochastic demand. Zhou (2007) studied four quantity-discount pricing policies – regular quantity, fixed percentage, incremental and fixed marginal-profit-rate discounts – in a single-producer single-retailer inventory problem in which stochastic price-sensitive demand is assumed. Burke et al. (2008) analyzed the impact of supplier pricing schemes and supplier capacity limitations on the optimal sourcing policy for a single firm in which the all-units discount is used. Mendoza and Ventura (2008) developed two EOQ models with transportation costs in which all-units and incremental quantity discounts are separately included. San-José and Garcia-Laguna (2009) developed a continuous review inventory problem with fully backlogging shortages and an all-units discount. Ebrahim et al. (2009) developed a multi-objective integrated supplier selection and inventory control problem in which all-unit, incremental and total-business-volume discounts are considered. Munson and Hu (2010) considered four different situations based on centralized and decentralized pricing and purchasing systems with local distribution in which both all-units and incremental discounts are included. Bai and Xu (2011) considered an inventory problem with dynamic demands in which the retailer may replenish its inventory from several suppliers, each of which is characterized by one or more than one type of order cost structure: incremental quantity discount, multiple setups, and all-unit quantity discount cost structures. Du et al. (2013) studied the coordination of two-echelon supply chains using wholesale price discount and credit options. Hong and Lee (2013) developed an optimal time-based consolidation policy with price sensitive demand and all-units discount which is used for dispatch cost.

Beginning with Montgomery et al. (1973), a number of authors have developed models for the basic EOQ with partial backordering. Pentico and Drake (2011) surveyed deterministic EOQ and EPQ models that include partial backordering. Taleizadeh et al. (2012) and Taleizadeh and Pentico (2013a) developed EOQ models with partial backordering under a special short-term sale and permanent known price increase respectively. Taleizadeh et al. (2013b) and Taleizadeh et al. (2013c) developed EOQ models with partial backordering under partial delayed payment and advanced payment, respectively.

The only previous models that include both all-units quantity discounts and partial backordering were by Wee (1999), Papachristos and Skouri (2003) and Ouyang et al. (2008). In contrast with what we will do in this paper, all three of these models included other complicating factors in addition to partial backordering and quantity discounts. After describing the additional ideas incorporated in these models, we will discuss how their primary assumptions differ from ours.

Wee's (1999) model included product deterioration at a constant rate and considered price to be a decision variable, with demand being a linear function of the price. Unlike almost all the partial backordering models in the literature, he also assumed that the length of an inventory cycle is an integer. Papachristos and Skouri (2003) extended Wee's scenario by using Weibull-distributed deterioration and a demand rate that is a convex decreasing function of the price. Ouyang et al.'s (2008) included non-instantaneous deterioration (i.e., there is an initial period during the cycle with no deterioration) and assumed that demand is a positive linear function of the inventory level rather than a function of the price.

In this paper we propose a deterministic EOQ model with partial backordering at a constant rate in which an all-units discount is used. All the other basic assumptions of the basic EOQ are made. As in Pentico and Drake's (2009) model for the basic EOQ with partial backordering and no quantity discounts, the decision variables are T, the length of an inventory cycle, and F, the percentage of demand that will be filled from stock or the percentage of a cycle during which there is positive inventory. Using the values of these two decision variables, the optimal order, backordering and lost sales quantities can be easily determined. In addition to the model characteristics mentioned above, the models developed by Wee (1999), Papachristos and Skouri (2003) and Ouyang et al. (2008) differ from ours as follows:

- *Cycle timing*: all three models use T, the cycle length, which is the same as ours, and T_1, the length of time for which there is positive inventory, which is the same as our $F \times T$. Papachristos and Christi treat T as a parameter, not a decision variable. Wee and Ouyang et al. treat T as a decision variable, but Wee assumes it is an integer and Ouyang et al. assume it is continuous.
- *Backordering rate*: as we do, Wee assumes the backordering rate is a constant. Ouyang et al. assume that it is stochastic, but since they only use the mean of the distribution, it is effectively a constant. Papachristos and Christi assume that it is a deterministic function of the time until the order can be filled.
- *Lost sale cost per unit*: as in most inventory research, our assumption is that this is the sum of a constant term for the good-will loss and the lost profit for the unsold unit. Wee and Ouyang et al. also assume that it has two parts, but they use the good-will loss and the lost *revenue* per unit, not the lost profit. In Papachristos and Christi it is a constant, independent of the cost or the sales price. In addition, Papachristos and Christi point out that Wee's revenue function is based on the revenue for all demand, not just the demand that is filled, thus ignoring the effect of lost sales.

Although incorporating additional situational characteristics, such as deterioration and a price-, time- or inventory-level-based demand function, makes for interesting models, they are considerably more complicated to solve and the user tends to lose the benefit of seeing what the impact of the discounts and partial backordering are.

Our proposed procedure and a numerical example to illustrate its application are given in Sections 2 and 3 respectively. In order to gain some insight into the effectiveness of this procedure relative to two possible alternative models – the basic EOQ without backordering and the EOQ with full backordering, we conducted a multi-factor experiment. The experiment and a discussion of its results are the subject of Section 4.

2. Model development

In this section we develop a model for the EOQ with all-units quantity discounts and partial backordering at a constant rate β. We will then see how, by setting $\beta = 1.0$, we can use this model to solve the EOQ with all-units quantity discounts and full backordering. But first we briefly review Pentico and Drake's (2009) deterministic EOQ models with full and partial backordering when no quantity discount is assumed.

The notation in Table 1 is used to model the problem.

2.1. EOQ models with no discount

Pentico and Drake (2009) derived the optimal values of F and T for the EOQ model with fully backordered shortages, shown in Fig. 1, as

$$T^* = \sqrt{\frac{2A}{iCD}}\sqrt{\frac{\pi + iC}{\pi}} \tag{1}$$