Omega 41 (2013) 510–516

Contents lists available at SciVerse ScienceDirect

Omega

journal homepage: www.elsevier.com/locate/omega

Multi-product lot scheduling with backordering and shelf-life constraints

Changyuan Yan [a], Yi Liao [b,*], Avijit Banerjee [a]

[a] Drexel University, Philadelphia, PA, USA

[b] Southwestern University of Finance and Economics, Department of Information Systems and Supply Chain Management, School of Business Administration, 555 Liutai Avenue, Chengdu, Sichuan Province 611130, China

ARTICLE INFO

Article history:
Received 23 September 2010
Accepted 21 June 2012
Processed by B. Lev
Available online 1 July 2012

Keywords:
Economic lot scheduling
Shelf-life constraint
Backordering

ABSTRACT

In this paper, we revisit the economic lot scheduling problem (ELSP), where a family of products is produced on a single machine, or facility, on a continual basis. Our focus is on the determination of a feasible production schedule, including the manufacturing batch size of each item. We assume that total backordering is permissible and that each of the products has a limited post-production shelf life. Several studies examining this problem have suggested a rotational common cycle approach, where each item is produced exactly once every cycle. To ensure schedule feasibility, we resort to the technique of reducing individual production rates and allow the flexibility of producing any item more than once in every cycle, in conjunction with appropriate timing adjustments. In order to solve this more generalized model, which is NP hard, we suggest a two-stage heuristic algorithm. A numerical example demonstrates our solution approach.

1. Introduction

This paper examines the problem of determining the optimal production rate, manufacturing batch size and the production frequency for each item within a family of products that are processed on a single machine, or within a single capacitated facility. Total backordering is allowed for any of the products, each of which has a specified, finite post-production shelf-life. Our objective is to determine a feasible production schedule for these items, while attempting to minimize the total relevant cost pertaining to the entire family of products.

The economic lot scheduling problem (ELSP) with shelf-life constraints has been examined earlier by Silver [9,10], Sarker and Babu [8], Goyal [1], Viswanathan [13], Viswanathan and Goyal [14,15], and Sharma [3,4,5,6,7]. Silver [9] incorporates the characteristics of shelf-life constraints in his rotational cycle model and discusses two ways to satisfy these constraints. One approach involves slowing down the production rates and the other reduces the production cycle time. Silver [9] proves that slowing down the production rates is a more effective technique. Sarker and Babu [8], on the other hand, analyze the same model outlined by Silver [9] and show that reducing the cycle time sometimes can be more effective, if a machine or facility operating cost is considered. Subsequently, Silver [10] deals with the shelf-life constraints in cyclic scheduling, by adjusting both the cycle time and the items' production rates. He specifically examines the situation where the cost-minimizing cycle time leads to the violation of one of the shelf-life constraints. Along similar lines, Viswanathan and Goyal [14] develop a model and provide an algorithm for determining the optimal production rate for each item and the optimal cycle time for the product family. Furthermore, Viswanathan and Goyal [15] embellish their earlier model by allowing backorders. In recent years, this problem has received significant research attention. For example, papers by Sharma [3,4] incorporate shortages and fractional backordering, respectively; also, Sharma [5] includes a generalized production cost; and Sharma [6,7] focus on the estimation of the inventory carrying cost and its modification. Recent studies have focused on the development of optimal and heuristic solution techniques for the ELSP. Notably, Grznar and Riggle [2] provide a global optimum solution for the basic period approach to the ELSP, whereas Tempelmeier [11] develops a column generation heuristic to deal with the dynamic ELSP under stochastic demands with a service level constraint.

All the studies mentioned above, however, assume that each of the items in question is produced exactly once in every rotational manufacturing cycle. Goyal [1] points out that producing some of the items more than once in a cycle may be more cost-effective. He assumes specific values of the production (or setup) frequency of each item and determines the minimum total relevant cost, given the number of batches per cycle for each item. Nevertheless, Viswanathan [13] points out that this approach may sometimes lead to infeasible schedules. Furthermore, it is assumed that the setup frequencies for the items are known *a priori*. These two earlier studies do not explore the derivation of the appropriate number of setups for each item in a cycle.

* Corresponding author. Tel.: +0118615184399766.
E-mail address: yiliao@swufe.edu.cn (Y. Liao).

0305-0483/$ - see front matter
http://dx.doi.org/10.1016/j.omega.2012.06.004

In this paper, we develop an extended model for the ELSP, allowing for backorders and limited post-production shelf-lives for the items in the family, while removing a restrictive assumption made in earlier studies and allow each of the items to be produced more than once in every cycle. Based on our model, we attempt to determine the corresponding optimal production policy, including each item's production frequency, its lot size, as well as its manufacturing rate and a feasible overall production schedule. The distinguishing feature of our work, compared to existing studies, is that the number of setups for each item in a rotational production cycle is treated as a decision variable and is determined through the solution of our model. We illustrate the model developed and its solution, in order to indicate the efficacy of our approach, via a numerical example.

2. Assumptions and notation

2.1. Assumptions

We make the following major assumptions in developing our model:

(a) The demand rate for each item is known and constant.
(b) The setup time for each item is known and constant.
(c) Inventory transactions are based on the FIFO rule.
(d) Total backordering is allowed for each item.
(e) Each item has a limited post-production shelf life.
(f) There is a machine operating cost per time unit.
(g) The production rate of each item is treated as a decision variable.

The third assumption above, i.e. the FIFO rule, is needed for simplicity of analysis in dealing with items having finite shelf lives. Also, the assumption concerning the existence of a machine operating cost allows the possibility of cycle time reduction to be effective from a cost reduction standpoint.

2.2. Notation

Our model development process and subsequent analyses are based on the following notational scheme.

(a) For the entire family:

N total number of items;
T the production cycle time;
O the machine operating cost per unit time;
C the average total relevant cost per time unit.

(b) Parameters for item i ($i=1, 2,\ldots, N$)

d_i the item's demand rate;
$p_i^{\max}$ the item's maximum possible production rate;
$r_i^{\min}$ $=d_i/p_i^{\max}$;
h_i the item's unit holding cost per time unit;
b_i the item's unit backorder cost per time unit;
l_i post-production shelf-life;
ρ_i setup time per batch;
s_i setup cost per batch (excluding machine operating cost during setup).

(c) Variables for item i ($i=1, 2,\ldots, N$)

p_i the adopted production rate for the item;
r_i d_i/p_i;
f_i production frequency per cycle;
t_i T/f_i, the item's cycle time;
k_i the item's production start time;
α_i^j the start time advancement for item i in its jth production batch ($1\le j\le f_i$);
α_i $\sum_{j=1}^{f_i}\alpha_i^j$, the total start time advancement for item i over the entire production cycle;
β_i^j the start time delay for item i in its jth production batch ($1\le j\le f_i$);
β_i $\sum_{j=1}^{f_i}\beta_i^j$, the total start time delay for item i over the entire production cycle;
a_i an adjustment cost during the total production cycle time;
c_i the total relevant cost over the item's production cycle time.

3. Model development

As mentioned earlier, Viswanathan [13] points out that a policy allowing for more than one setup per cycle for the items may lead to an infeasible schedule. However, when this happens, the schedule can be adjusted by advancing or delaying the start time(s) of one or more of the items, in order to achieve feasibility. We demonstrate this by a simple example, where we have 2 items in a family, with setup frequencies per cycle of 1 and 2, respectively. As shown in Fig. 1, during time T to T′, both item 1 and 2 are scheduled for production simultaneously, which makes this schedule infeasible. In order to force schedule feasibility, we can advance the start time of the second batch of item 2 in the production cycle by T′–T, as shown in Fig. 2. An alternative approach would be to delay the start time of item 1 in the second production cycle by an amount T′–T, as shown in Fig. 3.

3.1. Cost function

If a schedule is adjusted for attaining feasibility, the appropriate adjustment costs should be considered. Based on our analysis shown in the Appendix, the total adjustment cost can

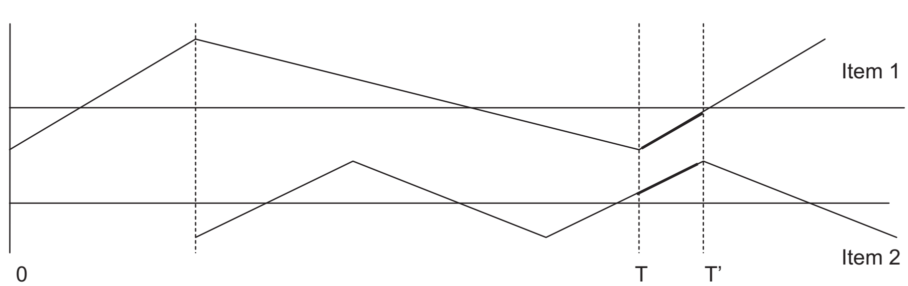

Fig. 1. Time conflict in original schedule.