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An EOQ model with partial backordering and advance payments for an evaporating item

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ABSTRACT

In the classic Economic Order Quantity model the purchasing cost of an order is paid at the time of its receipt. In some cases retailers ask purchasers to pay all or a fraction of the purchasing cost in advance and may allow them to divide the prepayment into several equal-sized parts. In this paper, an economic order quantity model for an evaporating item with partial backordering and partial consecutive prepayments is developed with a real case study of a gasoline station. Real numerical examples illustrate the proposed model and the solution method.

1. Introduction and literature review

Timing the payment of purchasing cost has a significant impact on the decision variables of inventory control systems. There are three different basic strategies for paying the ordering cost: (i) payment at the time of delivery, (ii) postponed payment or credit payment, and (iii) prepayment. A combination of two or three of these is also possible. In this paper we will focus on an economic order quantity model with partial backordering for an evaporating product when a fraction of the purchasing cost must be prepaid.

Inventory control models for deteriorating or perishable items have been widely investigated. After developing the EOQ model by Harris (1913), the first model for perishable products was developed by Ghare and Schrader (1963), in which the deterioration rate was assumed constant and the purchasing cost must be paid at the time of delivery. Under this payment strategy, Covert and Philip (1973) extended Ghare and Schrader's constant deterioration rate to a two-parameter Weibull distribution. This topic has subsequently been investigated by many researchers. Wee and Yu (1997) extended an EOQ model for deteriorating items when the vendors offers a temporary discount for the buyer. Taleizadeh et al. (2013b) developed an EOQ model with special sale and shortage for a deteriorating product. Soni (2013) studied optimal replenishment policies for non-instantaneous deteriorating items with stock and price sensitive demand where permissible delay in payment is permitted. Yu (2013) studied a collaborative strategy for deteriorating inventory system with imperfect items and supplier credits. Yang et al. (2013) investigated the collaboration for a closed-loop deteriorating inventory supply chain with multi-retailer and price-sensitive demand. Huang (2012) provided some comments for inventory control models for deteriorating products. Liao et al. (2012) studied lot-sizing decisions for deteriorating items with two warehouses under an order-size-dependent trade credit.

In the case of partial backordering, Wee (1995) studied a replenishment policy for a perishable product where demand declines exponentially, deterioration is assumed to be a constant fraction of on-hand inventory, and partial backordering of unfilled demand is assumed. Abad (2000a) developed a pricing and lot sizing problem for a vendor with a general deterioration rate, a general demand function and partial backordering. Abad (2000b) studied the problem of determining the lot size for a deteriorating product under finite production, exponential deterioration and partial backordering. Abad (2003) extended his previous work by studying the lot sizing and pricing problem for a deteriorating product in a production system with exponential decay and partial backordering and lost sales. Wee et al. (2005) studied a two-warehouse inventory control model for a perishable product with partial backordering and Weibull distributed deterioration. They considered inflation and applied the discounted cash flow in problem analysis. Yu et al. (2005) extended an inventory-production model for a perishable product with partial backordering and imperfect items. Law and Wee (2006) studied an inventory-production model from the perspectives of both retailer and manufacturer in which both deteriorating and ameliorating effects are taken account and time discounting and partial backordering are considered. Pal et al. (2006) developed a model to determine the lot size of a perishable product with the demand

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rate which depends on stock level, selling price and frequency of advertisement in which partial backordering is assumed. Teng et al. (2007) further developed the work of Abad (2003) by adding backlogging and lost-sale goodwill costs. Lo et al. (2007) developed an integrated two-layer inventory-production model for a perishable product in which partial backordering, inflation, imperfect production processes and multiple deliveries are assumed. Abad (2008) developed his previous work (Abad (2003)) by considering the EOQ model instead of the EPQ model. Chiao et al. (2008) studied an inventory model for deteriorating product with two storage facilities, quantity discount and partial backordering. They considered two storage facilities because of the limited capacity of the owned warehouse and the excess stock will be stored in the rented warehouse. The rented warehouse has a lower rate of deterioration but a higher unit holding cost. Gosh et al. (2011) developed an EOQ model for a single deteriorating product with price-dependent demand and partial-backordering which depends on the length of the waiting time for the next replenishment. Pang (2011) developed an integrated inventory-pricing model with dynamic pricing and periodic-review inventory policy, fixed ordering cost, and additive demand. The inventory deteriorates over the time and the unsatisfied demand will be partially backlogged. Jolai et al. (2006) developed an economic production lot size model for a perishable product with partial backordering, stock-dependent demand and inflation. A comprehensive survey on partial backordering appeared in Pentico and Drake (2011). Bakker et al. (2012) presented an up-to-date review of the advances made in the field of inventory management of deteriorating items from 2001 to 2011, while for before 2001, Goyal and Giri (2001) made a distinction between three broad categories of inventory in their excellent review. Sarkar (2012a) developed an EOQ model for a deteriorating product with time-varying deteriorating rate and delayed payment. Sarkar (2012b) extended an EOQ model with stock dependent demand in the presence of imperfect production and the delayed payment method. Sarkar and Sarkar (2013) developed an improved inventory model with partial backordering, stock-dependent demand and time varying deterioration rate. Sarkar and Moon (in press) studied an inventory model with improved quality, setup cost reduction, and variable backorder costs in an imperfect production process. Taleizadeh et al. (2012) developed an EOQ model with partial backordering and special sale. Taleizadeh and Pentico (2013) developed an EOQ model with partial backordering and known price increase. Taleizadeh et al. (2013c) developed an EOQ model with partial backordering and partial delayed payment. Taleizadeh and Pentico (in press) developed an EOQ model with partial backordering and all-units discount. Moreover a brief comparison of some other researches is shown in Table 1.

In the case of prepayment, Taleizadeh et al. (2011) developed a joint replenishment inventory control model for non-deteriorating items and partial prepayment in uncertain environment. Then Taleizadeh et al. (2013a) developed an EOQ model with partial backordering and partial prepayments. They assumed that the supplier asks purchasers to pay a fraction of the order's cost in advance and may allow them to divide the prepayment into multiple equal-sized parts to be paid during a fixed lead time. In this paper, an economic order quantity model for a deteriorating product with partial backordering and partial consecutive prepayments will be developed.

2. Problem definition

Consider a situation where a supplier asks his customers to prepay a fraction, α, of the purchasing cost when an order is placed, as the first prepayment at time L before the delivery of a lot. The supplier may ask that a prepayment is made to settle the balance or in multiple installments at equal intervals. In a prepayment purchasing system the capital cost of the costumer will increase because he/she has incurred interest costs on α percent of the purchasing cost of products which have not yet been received. Also it is assumed that n, the number of prepayments, offered by the supplier. In the following, an economic order quantity model for a deteriorating item with multiple prepayments and partial backordering is developed.

3. Model development

The parameters and variables of the model are introduced in Section 3.1 and the model is developed in Section 3.2.

3.1. Notation

The following notations are used to model the problem.

Parameters:

A	the fixed order cost ($/order)
α	the fraction of purchasing cost that must be paid as multiple advance payments
β	the partial back-ordering rate
C	the purchasing cost ($/unit)
C'	the marginal purchasing cost including capital cost of prepayment ($/unit)
D	the demand rate per month
h	the holding cost including evaporation cost ($/unit /period)
I	the maximum inventory level (unit)
I_c	the capital cost rate ($/unit/period)
θ	the evaporation rate
P	the selling price ($/unit)
g	the goodwill loss for a lost sale ($/unit)
L	the length of time during which the buyer will pay the prepayments (time)
n	the number of equally spaced prepayments to be made before receiving the order
π	the backorder cost ($/unit)
π'	the lost sale cost ($/unit), $\pi' = P - C' + g$
$I(t)$	the inventory level at time t
$B(t)$	the back-ordered level at time t

Decision variables:

B	the backordered quantity
F	the percentage of demand that will be filled from stock
Q	the order quantity
N	the number of inventory cycles
T	the length of an inventory cycle in periods, $N = 1/T$
(*)	indicates the optimal value

Other variables:

ATC	the annual total cost of the partial backordering case
ATP	the annual total profit of the partial backordering case
CCC	the cyclic capital cost
CTC	the cyclic total cost
CTP	the cyclic total profit

3.2. Modeling and solution method

Here the EOQ model with multiple partial prepayments and partial backordering for an evaporating product will be modeled. Consider a situation where the buyer uses an EOQ model to control the inventory of an evaporating product where the shortage is partially being backlogged. According to Fig. 1 the inventory level at time t decreases because of demand and evaporation. Thus the