Int. J. Production Economics 162 (2015) 151–159

Contents lists available at ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

Approximating the EOQ with partial backordering at an exponential or rational rate by a constant or linearly changing rate

David W. Pentico [a,*], Carl Toews [b,1], Matthew J. Drake [a,2]

[a] Palumbo-Donahue School of Business, Duquesne University, Pittsburgh, PA 15282-0180, United States
[b] Department of Mathematics & Computer Science, University of Puget Sound, Tacoma, WA 98416, United States

ARTICLE INFO

Article history:
Received 18 August 2014
Accepted 18 January 2015
Available online 31 January 2015

Keywords:
EOQ
Partial backordering
Approximate solutions

ABSTRACT

A significant extension of the classic EOQ model is the assumption that realized demand decreases if customers are forced to backorder. To capture the way this decrease depends on the waiting time, different functional forms have been proposed, ranging from the simple (e.g., constant or linear forms) to the complex (e.g., exponential or rational forms.) This paper explores the question of whether the computationally more tractable simple forms can give high quality approximations to the complex ones. We calculated average and worst case performance on a representative suite of test problems, each characterized by a "backorder resistance" parameter. We show that for low values of this parameter, results from the approximating functions are virtually as good as those from the correct ones, and for high values of the parameter, very good results can be achieved by using an iterative technique.

1. Introduction

The economic order quantity (EOQ) model first published by Harris (1913) is the foundation for many inventory models that are still being developed more than a century later. Countless inventory models have taken the original EOQ model and have relaxed one or several of its underlying assumptions. See Drake and Marley (2014) for a discussion of the major extensions of the EOQ model that have been published in each decade since 1950.

A significant set of publications have considered consumer behavior known as "partial backordering," where only a fraction of customers who are faced with a stockout situation are willing to wait for an order. Most of the models for the basic deterministic economic order quantity with partial backordering (EOQ-PBO) developed by Montgomery et al. (1973), Rosenberg (1979), Park (1982,1983), and Wee (1989), one of the models included in San José et al. (2005), and the recent model by Pentico and Drake (2009) made all the usual assumptions of the basic deterministic EOQ model with full backordering except that they assumed that a constant fraction β of the demand when there is no stock will be backordered, with the remaining fraction $1-\beta$ being lost sales. A smaller set of models, such as those by Mak (1987), Pentico et al. (2009), and San José et al. (2014), analyzed partial backordering in the context of a finite production rate, known as the economic production quantity (EPQ) model.

Montgomery et al. (1973) also briefly discussed a model for the EOQ-PBO with a backordering rate that changes over time according to a linear function $\beta(\tau)$, where τ is the time remaining until the backorder can be filled. More complete developments of this model are in San José et al. (2007) and Toews et al. (2011), which also considered the EPQ-PBO. Other than the one by Montgomery et al. (1973), the first models to include a backordering rate that increases with the time to delivery were developed by Abad (1996), who assumed that $\beta(\tau)$ is either an *exponential* or a *rational* function of τ. For both of these functions $\beta(\tau)$ increases as τ decreases, approaching its maximum value, which is usually assumed to be 1.0, when $\tau=0$.

With the exception of the model by Abad (1996), the models mentioned so far limited their extension of the basic EOQ model to include only partial backordering. There have been many other models that included, as Abad (1996) did, additional situational considerations, such as deteriorating or perishable inventory, pricing, quantity discounts, time-dependent holding costs, the demand level and pattern, warehousing, payment terms, or multiple items with correlated demand. Abad (2008) extended Abad (1996) by including shortage, backordering, and lost sale costs as well as pricing. Wee (1999) included deteriorating inventory, quantity discounts, and pricing, as did Papachristos and Skouri (2003). Interestingly, Mishra et al. (2013) developed a model for deteriorating items where the *holding cost* is time-dependent, but the backorder rate is constant. Taleizadeh and Pentico (2014) focused on a partial backordering

* Corresponding author: Tel: +1 412-396-6252.
E-mail addresses: pentico@duq.edu (D.W. Pentico), toewsc@gmail.com (C. Toews), drake987@duq.edu (M.J. Drake).
[1] Tel: +1 253–879-3839.
[2] Tel: +1 412-396-1959.

http://dx.doi.org/10.1016/j.ijpe.2015.01.014
0925-5273/

model with an all-units quantity discount. Lo et al. (2007) modeled an integrated buyer-production system in which the producer has an imperfect process, both the raw materials and finished product deteriorate, and all items are subject to price inflation. Yang (2006) and Yang (2012) both modeled two-warehouse systems in which there is inflation and product deterioration; Bera et al. (2013) developed a similar two-warehouse model with product deterioration, including demand that is dependent on time, price, and advertising frequency. Yang et al. (2010) considered inflation, product deterioration, and a stock-level dependent demand rate, while Maihami and Kamalabadi (2012) included pricing, deterioration, and demand that depends on both time and price. Taleizadeh et al. (2013) and Taleizadeh (2014) both modeled different payment terms such as prepayments or delayed payments. Brief discussions of these papers, except those published after 2011, may be found in Pentico and Drake (2011).

A significant problem with EOQ-PBO models in which $\beta(\tau)$ has any form other than a constant β or a linear function of τ is that they do not have a closed-form solution, even for models that do not include such complicating features as inventory deterioration or demand that depends on either time or the inventory level. Solving the models for any other form for $\beta(\tau)$ involves some sort of search process, usually either non-linear programming or some type of iterative procedure.

There are at least two problems with using these non-closed-form solution methods. First, they are more time-consuming and harder to automate. Second, and the primary impetus for the research reported here, is that they are more difficult for many, if not most, managers to understand. Why the difficulty of understanding how a model and/or its solution procedure works is a relevant issue for managing inventory was summarized succinctly by Woolsey and Swanson (1975): “People would rather live with a problem they cannot solve than accept a solution they cannot understand.” A broader rationale for using simpler, easier to solve models is given by a quote attributed to Albert Einstein, which is really a paraphrase of a more complicated statement he made: “Everything should be made as simple as possible, but no simpler.” That is: use as simple a model as possible that will do the job well. These two quotes provide the justification for what we are attempting here.

In this paper we will consider the accuracy of approximating the EOQ-PBO with a backordering rate $\beta(\tau)$ that is either an exponential or rational function of τ, the time remaining until the backorder can be filled, by the EOQ-PBO with a either a constant β or a linear $\beta(\tau)$. Assuming that either or both of these approximation methods gives high quality results, then either or both of them can serve as the basis for approximation methods for more complicated scenarios with a non-linear backordering rate function, such as those including deterioration or time- or stock-level-based demand.

2. Summaries of the partial backordering models

The models we will use for the EOQ with partial backordering are those in Pentico and Drake (2009) for the EOQ-PBO with a constant β, Toews et al. (2011) for the EOQ-PBO with a linear function for $\beta(\tau)$, San José et al. (2006) for the EOQ-PBO with an exponential function for $\beta(\tau)$ and San José et al. (2005) for the EOQ-PBO with a rational function for $\beta(\tau)$.

2.1. The constant-β and linear-$\beta(\tau)$ models

The notation for the parameters and variables to be used, which is basically the same as the notation used in Toews et al. (2011), is given in Table 1.

2.1.1. The EOQ-PBO with a constant backordering rate β: Pentico and Drake (2009)

The model in Pentico and Drake (2009) makes all the assumptions of the basic EOQ with full backordering model except it assumes that only a given fraction β of the demand during the time that the system is out of stock is backordered, with the complementary fraction $1-\beta$ being lost sales. The average cost per period is as follows:

$$C(T,F) = \frac{C_o}{T} + \frac{C_h DTF^2}{2} + \frac{\beta C_b DT(1-F)^2}{2} + C_l D(1-\beta)(1-F) \quad (1)$$

The values of T and F that minimize the average cost per period are as follows:

$$T^* = \sqrt{\frac{2C_o}{DC_h}\left[\frac{C_h+\beta C_b}{\beta C_b}\right] - \frac{[(1-\beta)C_l]^2}{\beta C_h C_b}}. \quad (2)$$

$$F^* = F(T^*) = \frac{(1-\beta)C_l + \beta C_b T^*}{T^*(C_h+\beta C_b)} \quad (3)$$

only if β satisfies the condition given by:

$$\beta \geq \beta^* = 1 - \sqrt{2C_0 C_h D}/(DC_l) \quad (4)$$

and $C(T^*,F^*) = C_h DT^*F^* < C_l D$, the cost of not stocking the item.

2.1.2. The EOQ-PBO with a linear function for $\beta(\tau)$: Toews et al. (2011)

The model in Toews et al. (2011) makes the same assumptions as the model in Section 2.1.1 except that it assumes that $\beta(\tau)$ is a

Table 1
Symbols used in the constant-β and linear-$\beta(\tau)$ models.

Parameters
D=Demand per period
s=The unit selling price
C_o=The fixed cost of placing and receiving an order
C_p=The variable cost of purchasing a unit
C_h=The cost to hold a unit in inventory for a period
C_b=The cost to keep a unit backordered for a period
C_g=The goodwill loss on a unit of unfilled demand
$C_l=(s-C_p)+C_g$=The cost for a lost sale, including the lost profit on that unit and any goodwill loss
β=The fraction of stockouts that will be backordered in a constant backordering rate model
β_0=The initial fraction of stockouts that will be backordered in a linear backorder rate model
$\beta(\tau)$=The fraction of stockouts that will be backordered at time τ in a linear backorder rate model
τ=The time until the backorder will be filled
Variables
T=the length of an order cycle
F=the fill rate or the percentage of demand that will be filled from stock