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Analysis of an EOQ inventory model with partial backordering and non-linear unit holding cost ☆

L.A. San-José [a,*], J. Sicilia [b], J. García-Laguna [c]

[a] IMUVA, Departamento de Matemática Aplicada, Universidad de Valladolid, Valladolid, Spain
[b] Departamento de Matemáticas, Estadística e Investigación Operativa, Universidad de La Laguna, Tenerife, Spain
[c] IMUVA, Departamento de Estadística e Investigación Operativa, Universidad de Valladolid, Valladolid, Spain

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ABSTRACT

In this paper, an economic order quantity inventory model is analyzed, considering that the unit cumulative holding cost has two significant components: a fixed cost which represents the cost of accommodating the item in the warehouse and a variable cost given by a potential function of the length of time over which the item is held in stock. Shortages are allowed and, during the stockout period, only a fraction of demand is partially backordered. The backordering cost includes a fixed cost and a cost linearly dependent on the length of time for which backorder exists. A solution procedure is developed for determining the optimal inventory policy. Moreover, to illustrate the effects of some parameters on the optimal policy and the minimum total inventory cost, a numerical study is developed.

1. Introduction

The Economic Order Quantity (EOQ) model proposed by Harris [16], and subsequently by Wilson [48], was the first lot-size inventory model based on cost minimization. In this EOQ model, shortages are not allowed and the inventory cost is the sum of the costs of purchasing, ordering and holding of the items in stock. Moreover, the holding cost for any item held is a linear function of the length of time over which the item is stored. For many years, the hypothesis of linearity of the holding cost was considered a necessary simplification for a comfortable formulation of the model (see, e.g., [15], p. 14).

Several extensions of this EOQ model have been developed for dealing with other expressions of the holding cost. Thus, some researchers introduced the non-linearity of the holding cost in the inventory models developed by them. Naddor [23] studied a model where the holding cost is non-linear with respect to time, another one with non-linear holding cost in relation to the quantity ordered and, finally, a mixed model of the above models. The first of these models was named the lot-size inventory model for perishable items (see, e.g., [6], p. 122–123). Muhlemann and Valtis-Spanopoulos [22] developed a model which takes into account a possible disproportionate change in holding cost as the average value of stock increases. Weiss [47] considered two inventory systems with a potential holding cost function. He indicated that such models can be applicable to any inventory system where the value of the item decreases non-linearly the longer it is held in inventory. Goh [14] studied the integration of a non-linear holding cost in the presence of an inventory-level dependent demand rate. He analyzed two different ways to represent the holding cost, one by using a non-linear function of the length of time that the item is held in stock, and another considering a non-linear function of the amount of on-hand inventory. Giri and Chaudhuri [13] extended the previous model of Goh [14] to study an inventory system for deteriorating items, when the rate of deterioration is a constant fraction of the inventory level. Ferguson et al. [11] considered the problem studied by Weiss [47] and showed how it is an approximation of the optimal order quantity problem for perishable goods. Alfares [1] developed the inventory policy for an item with a stock-level dependent demand rate and a storage-time dependent holding cost. Urban [45] generalized the Alfares model to stock-dependent demand models with variable holding costs by using a profit-maximization objective. Pando et al. [24,25] solved the previous models of Goh [14] from the perspective of maximizing profit. Shah et al. [35] considered an inventory system with non-instantaneous deteriorating item and an arbitrary holding cost rate, in which demand rate is a function of advertisement of an item and selling price. Recently, Pando et al. [26] studied an inventory system with stock-dependent demand rate from the perspective of maximizing profits per unit time, assuming a holding cost which is non-linear with respect to both time and stock-level.

When shortages occur in the inventory system, two costs could be considered in the inventory control: the backorder cost and the

☆This manuscript was processed by Associate Editor Kovalyov.
* Corresponding author.
E-mail addresses: augusto@mat.uva.es (L.A. San-José), jsicilia@ull.es (J. Sicilia), laguna@eio.uva.es (J. García-Laguna).

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lost sale cost. In this paper, we assume a fixed lost sale cost, and a non-decreasing function of the waiting time for the backorder cost. More concretely, we suppose that the cost of a backorder is the sum of a fixed cost plus a cost linearly dependent on the length of time for which the backorder exists. This kind of backordering cost is also used by several authors, e.g., San-José et al. [31,32], Cárdenas-Barrón et al. [4], Chung and Cárdenas-Barrón [9], Sphicas [36] and Wee et al. [46]. Other researchers have also considered different structures for the backordering cost. Thus, Hu et al. [17] proposed a model in which the backorder cost per unit time is a linearly increasing function of shortage time. San-José et al. [30] studied an inventory system with partial backlogging where the unit backorder cost is a non-decreasing, continuous and positive function of the waiting time.

Some inventory models that allow shortages assume that the unsatisfied demand during a stockout period is completely backordered. There are several papers on inventory models assuming a time-dependent holding cost and full backlogging. Benkherouff [3] examined the problem of finding the optimal replenishment policy for an inventory model that minimizes the total expected discounted costs where the holding cost takes a general form. Bayindir et al. [2] presented a deterministic production model with shortages under a general inventory cost function. Sazvar et al. [34] developed an inventory model for perishable products considering non-linear cumulative holding cost and assuming that shortages are completely backordered. Ghasemi and Nadjafi [12] presented two EOQ models where the holding cost is an increasing continuous function of the ordering cycle length. In the first model, shortages are not allowed and, in the second model, shortages are completely backordered. Choudhury et al. [7] developed an EOQ model for deteriorating items where the holding cost is expressed as a linearly increasing function of time and shortages are fully backlogged.

However, in many real inventory systems, demand can be partially captive. That is, during the stockout period, some customers, whose needs are not critical at that time, can wait for the next replenishment to be satisfied; other customers, however, cannot or do not wish to wait and they have to fill their demand from another source. This behavior is captured by the use of partial backordering in the model formulation. In this line, there are the papers of Montgomery et al. [21], Rosenberg [29], Park [27], Mak [19], Chu and Chung [8], Yang [50], Leung [18], Pentico et al. [28], Zhang [51], San-José et al. [31], [32], Cárdenas-Barrón et al. [5], Drake et al. [10], Zhang et al. [52], Taleizadeh et al. [42], Stojkovska [37], Taleizadeh and Pentico [40], Taleizadeh et al. [43,44], Taleizadeh [38,39] and Taleizadeh and Pentico [41]. More concretely, in all of these papers, the authors consider that a fixed fraction of the demand is served late during the period without existences. Moreover, they developed the inventory models considering that the holding cost is a linear function of the average inventory.

Relatively few papers have considered models with partial backlogging and non-linear holding cost. Mishra et al. [20] developed a deterministic inventory model in which the holding cost is proportional to time and the demand is partially backlogged. Yang [49] studied a replenishment problem with partial backlogging where the demand rate and the holding cost depend on the stock level. Table 1 summarizes the major characteristics of the previously cited papers that have been published from the year 2000.

In the present paper, we study a deterministic EOQ model for a single item, allowing partial backlogging and assuming a unit holding cost which depends on storage time. More specifically, we suppose that the backlogged demand rate, at any instant, is a constant fraction and we consider that the holding cost per unit is the addition of a constant and a potential function of the length of time over which the item is held in stock. According to Weiss [47], we will analyze an inventory system where the value of the item decreases non-linearly the longer it is held in stock. This work extends the inventory systems studied by Montgomery et al. [21], Rosenberg [29] and Park [27]. Also, it modifies the deterministic model of Weiss [47] in three directions. First, we relax the assumptions imposed on the holding cost function. So, it is supposed that the unit cumulative holding cost has two significative components: a fixed cost, which represents the cost of accommodating the item in the warehouse, and a variable cost given by a potential increasing function of the length of time over which the item is held in stock. Second, shortages are allowed and only a fixed fraction of demand during the stockout period is backordered. Third, the backorder cost and the lost sale cost in the total cost function are considered. Moreover, the backorder cost includes a fixed cost and a cost proportional to the length of time for which backorders exist.

The goal is to minimize the total cost function of the inventory system throughout the inventory cycle. This cost function depends on two decision variables: the time period in which the net stock is positive and the time period where shortages occur in the system. We will obtain the optimal policy following a sequential optimization procedure with two stages. Thus, we will firstly optimize a univariate convex function (the other variable is fixed) and, in the second stage, we will determine the optimal value of a piecewise function depending solely on the other variable.

This paper is organized as follows. Section 2 summarizes the assumptions which characterize the inventory system under study, describes the notation used in the paper and presents the formulation of the considered model. Section 3 discusses the possible cases that can occur in order to determine the optimal solution of the problem. Numerical examples and sensitivity analysis are presented in Section 4. Finally, the conclusions are described in Section 5.

2. Description and formulation of the problem

In this section, we formulate the mathematical model for an inventory system with time-dependent holding cost and partial backlogging. We consider that only a fraction of demand is backlogged and the holding cost per unit includes a fixed cost and a time-dependent cost, which potentially increases with the time that the item is held in stock.

2.1. Assumptions

We assume the following conditions for the system under study. The item is a single product with independent demand and the planning horizon is infinite. The inventory system has a uniform demand pattern and the demand rate is known and constant. The inventory is continuously reviewed and the replenishment is instantaneous. The order cost is fixed regardless of the lot size. The unit holding cost is a non-linear and convex function of time in storage. The model allows shortages, which are partially backlogged. The fraction of backlogged demand is a fixed constant. The cost of a backorder includes a fixed cost and a variable cost, which is a linear function of the length of time for which the backorder exists. Finally, the cost of a lost sale is constant.

The fluctuations of the inventory level during an inventory cycle for our model are depicted in Fig. 1.

2.2. Notation

We adopt the following notation.

T length of the inventory cycle where the net stock is positive (≥ 0)
Ψ length of the inventory cycle over which net stock is less than or equal to zero (≥ 0)