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An EOQ model with partial delayed payment and partial backordering

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ABSTRACT

In many transactions concerning selling and buying, a specified delay of payment is offered or accepted by the seller. This can be regarded as a kind of discount and has potential consequences for the order size. These kinds of effects are not explicitly incorporated in the classical formulas for economic order quantities (EOQ). In this research we consider an EOQ problem under partial delayed payment. A fraction of the purchasing cost must be paid at the beginning of the period and the remaining amount can be paid later. Shortages are permitted and occur as a combination of backorders and lost sales. The aim of this paper is to determine the order and shortage quantities.

1. Introduction and literature review

The first major component of our model is partial delayed payments. When a delay in payment is possible, the capital cost of the purchaser is decreased, although the other costs are not. Several researchers have studied the effect of delayed payments on the EOQ. Goyal [1] was the first to develop a model for a delay in payment to the supplier, making all the usual assumptions of the classic EOQ model except for when payment is due. Aggarwal and Jaggi [2] developed an ordering policy for deteriorating items. Shinn et al. [3] proposed a model to simultaneously determine the retailer's optimal price and lot size. Hwang and Shinn [4] dealt with pricing and lot-sizing decisions for exponentially deteriorating products. Liao et al. [5] studied an inventory model with an initial-stock-dependent consumption rate. They assumed shortages are not allowed and the effects of inflation, deterioration, and the initial-stock-dependent consumption rates are discussed. Sarker et al. [6] developed a model to determine an optimal ordering policy for deteriorating items under inflation and fully backordered shortages. Chang and Dye [7] developed a model for the optimal ordering policy for deteriorating items when partial backordering is assumed. In all this research two common cases are considered; the delayed payment is due before the inventory is exhausted or after the inventory level reaches to zero.

Other authors have added additional considerations relative to the payments. Shinn and Hwang [8] discussed the situation of order-size dependent payment delays. Ouyang et al. [9] considered a variation on Chang and Dye's [7] problem and extended the analysis to four possible cases related to the delayed payment time and the issue of how the buyer pays for the order: (1) paying only for the goods sold at the deadline, paying for the rest when they are all sold, or (2) paying for the entire order, whether all sold or not, at the deadline.

Huang [10] assumed the supplier would offer the retailer a partial delay in payments when the order quantity is smaller than a predetermined quantity. Under this condition, he modeled the retailer's inventory system as a cost minimization problem to determine the retailer's optimal inventory cycle time and optimal order quantity. He assumed the demand rate is constant and shortages are not permitted. Jaggi et al. [11] proposed a model in which demand is linked to the credit period offered by the retailer to the customers. Sana and Chaudhuri [12] modeled the retailer's profit-maximizing strategy when confronted with its supplier's trade offer of credit and a price discount on the purchase of merchandise. In their model increasing deterministic demands are discussed, shortages are not permitted, and the supplier offers different discounted prices at different delay periods. Ho et al. [13] developed an integrated supplier–buyer model with the assumption that the market demand is sensitive to the retail price and the supplier adopts a trade credit policy. The trade credit policy discussed in their research is a "two-part" strategy: cash discount and delayed payment. Chu et al. [14] modified

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the results in Aggarwal and Jaggi [2] by showing that the objective function is piecewise convex, which enabled them to improve the solution procedure. Tsao and Sheen [15] investigated the problem of dynamic pricing, promotion and replenishment for a deteriorating item subject to the supplier's trade credit and retailer's promotional effort. They adopted a price- and time-dependent demand function to model the finite-time-horizon inventory for deteriorating items. The objective of their problem is to determine the optimal retail price, the promotional effort and the replenishment quantity so that the net profit is maximized. Chung and Liao [16] discussed the optimum order quantity of an EOQ model that is not only dependent on the inventory policy but also on the firm's credit policy. They incorporated the concepts of a discounted cash-flows (DCF) approach, trade credit and the quantity ordered and developed a new inventory model with a delay in payment. Ouyang et al. [17] generalized an economic order quantity (EOQ) model with a permissible delay in payment to reflect the following conditions: (1) the retailer's selling price per unit is significantly higher than unit purchase price, (2) the interest rate charged by a bank is not necessarily higher than the retailer's investment return rate, (3) the supplier may offer a partial delay in payments even if the order quantity is less than the quantity at which the fully delayed payment is permitted. They assumed the demand rate is constant and shortages are not permitted. Chen and Kang [18] developed an integrated supply chain model for determining the optimal replenishment time interval and replenishment frequency. Their model enabled them to develop a simple algorithm for allocating the cost savings between the vendor and the buyer. Chen and Kang [19] developed integrated inventory models in which customers' demand is sensitive to the price. Kreng and Tan [20] developed a model to determine the optimal replenishment decisions under two levels of trade credit policy if the purchasers' order quantity is greater than or equal to a predetermined quantity. Teng et al. [21] developed an economic order quantity model with delayed payment strategy and non-decreasing demand. Chung [22] indicated the incompleteness of two theorems that Teng et al. [21] used to derive the optimal solutions and corrected them.

The second major component of the model we develop here is partial backordering of demand that cannot immediately be filled from stock. The first paper that developed a model that made all the usual assumptions of the classic EOQ model with the addition of partial backordering of demand during a stockout period was by Montgomery et al. [23]. Several other authors have since developed comparable models, which are described in a survey by Pentico and Drake [24], along with other deterministic models that include not only partial backordering but a variety of other considerations, such as a time-dependent backordering rate, deteriorating inventory and time, price, or inventory-level-dependent demand patterns. The partial backordering model that is the basis for the model we present here was developed by Pentico and Drake [25]. While the early papers on the basic EOQ with partial backordering modeled the problem using Q, the order quantity, and S, the maximum stockout level, Pentico and Drake [25] used T, the time between orders or the inventory cycle length, and F, the fill rate or the percentage of demand filled from stock. In addition to simplifying the modeling and solution, this approach also made it possible to prove the optimality of the solution, which was not possible for some of the models developed in the earlier papers surveyed in Pentico and Drake [24]. Pentico et al. [26] extended their model for the EOQ with partial backordering in [25] to add the continuous delivery assumption of the EPQ model and showed that, with appropriate redefinition of the unit holding, backordering, and lost sale costs, the equations for the EOQ and EPQ with partial backordering are identical. Hsieh and Dye [28] developed a method for solving the model in Pentico et al. [26] based on the arithmetic–geometric mean inequality theorem instead of differential calculus. Pentico et al. [27] extended the model in Pentico et al. [26] to allow for the possibility that the percentage of demand backordered will increase when production starts again. Wee and Wang [29] extended the model in Pentico et al. [27] to allow the backordering rate to change at times other than when production starts.

2. Problem definition

Inventory holding as a tactical-level decision designed to increase the confidence level of responding to customer demands and to protect against all kinds of non-deterministic situations in receiving raw materials has made inventory control and management an important concept in supply chain management. The model that is proposed in this research is based on the real constraints and environments of manufacturing companies and resellers. As in the classic EOQ model, the demand and all other parameters are assumed to be deterministic and constant over an infinite horizon. The supplier requires that a specified fraction of the purchasing cost must be paid at the time of receipt and the remaining amount can be paid at a specified later date. As in other research on the EOQ with a permissible payment delay, we assume that sales revenue generated during the credit period is deposited in an account that earns interest, which we assume is at a rate that is less than the capital interest charge. Finally, we assume that shortages are permitted and result in a combination of backorders and lost sales. As in Pentico and Drake [25], we assume that the percentage of shortages backordered is constant over time.

In the next section we model the problem.

3. Problem modeling

To model the problem, we first introduce the parameters and the variables of the problem in Section 3.1. The model is then developed in Section 3.2.

3.1. Notation

Parameters:

α The fraction of the purchasing cost to be paid at the time of receipt.
A Fixed cost to place and receive an order
β The fraction of shortages that will be backordered
C Unit purchase cost of an item
D Demand quantity per year