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Optimal inventory management with supply backordering

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ABSTRACT

We study the inventory control problem of a retailer working under stochastic demand and stochastic limited supply. We assume that the unfilled part of the retailer's order is fully backordered at the supplier and replenished with certainty in the following period. As it may not always be optimal for the retailer to replenish the backordered supply, we also consider the setting in which the retailer has a right to either partially or fully cancel these backorders, if desired. We show the optimality of the base-stock policy and characterize the threshold inventory position above which it is optimal to fully cancel the replenishment of the backordered supply. We carry out a numerical analysis to quantify the benefits of supply backordering and the value of the cancelation option, and reveal several managerial insights.

1. Introduction

With sourcing in remote offshore locations, uncertainties in supply have increased. For instance, in 2009, during the recovery period of the 2008 financial crisis, electronics parts were in short supply and suppliers could not deliver orders. For instance, Nortech issued a formal statement in March 2009 stating that it was "*dependent on suppliers for electronic components and may experience shortages, cost premiums and shipment delays that would adversely affect our customers and us*" (Nortech Systems, 2009). In such cases, backorders are delivered very late. With regard to the same period of electronic component shortages in 2009 and 2010, Pierson (2010) reports that lead times for certain components had increased from 10 to 20 weeks, and that customers were on allocation, implying that customers were unsure how much they would receive. However, many buyers in fact later canceled their orders when they obtained new information on demand. The very late supply of backorders actually granted them a moral right to cancel the orders due to the very late delivery. When demand is uncertain, it is unclear to what extent it is best to actually cancel backorders and possibly place new orders instead.

In addition, the importance of time as a competitive weapon in supply chains has led suppliers to venture into lead time reduction projects to improve their ability to meet the customers' expectations for shorter lead times. However, such undertaking often results in, at least in a short term, worse supply performance, characterized mainly by delayed and/or partial replenishment of orders. Furthermore, a supplier might even decide to adopt the supply strategy where he would deliver only a part of the customer's order depending on his current supply capacity availability, and would guarantee to replenish the rest of the order with a short delay. From a customers' perspective, apart from the demand uncertainties, companies need to consider the possibility of these delays in deciding when and how much to order from the supplier. Interestingly, in a stochastic lead time setting, Wang and Tomlin (2009) show that a customer may sometimes prefer a less reliable lead time if the delay distribution is not very variable. Therefore, it may be beneficial for a company to accept the possibility of the uncertain delivery and, given that the order is eventually replenished in full, adapt its ordering policy to take advantage of a shorter lead time.

In this paper we study the inventory control problem of a retailer working under stochastic demand from the market, where he tries to satisfy the demand by making orders with a supplier. The supply capacity available to the retailer is assumed to be limited and stochastic as a result of a supplier's changing capacity and capacity allocation policy. The order placed by the retailer might therefore not be delivered in full, depending on the currently available capacity. The unfulfilled part of the retailer's order is backordered at the supplier, which we denote as a supply backorder. As the supply backorder is a result of the supplier's inadequate supply service, this gives the retailer an option (a moral right) to decide to what extent he wants the supplier to replenish the backordered supply. Depending on the current requirements, the retailer can decide for partial replenishment of the backordered supply, or to fully cancel the replenishment if necessary. Therefore, in each period the retailer has to make two decisions. Apart from the regular ordering decision to the supplier, he needs to decide about the extent of the replenishment of the

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backordered supply from the previous period. We assume that the replenishment of the backordered supply is certain, meaning that it is delivered in full in the following period. The situation that would result in such supply conditions for the retailer is that of a supplier giving high priority to fulfilling backorders from previous periods, which is a common situation observed in practice. It is also safe to assume that this would often not affect the regular capacity available to cover retailer's order in the next period.

The focus of this paper is on establishing the optimal inventory policy that would allow the retailer to improve his inventory control by utilizing a backordered supply. We denote the case where the backordered supply can be partially replenished at the request from the retailer as the *Partial backordering* policy, and compare it to the *No backordering* policy, where there is no supply backordering, to establish the value of supply backordering. Apart from this base setting, we are interested in analyzing the two subpolicies: *Full backordering* policy, where the backordered supply is always replenished in full, and the *Cancelation option* policy where the retailer has the option to fully cancel the replenishment of the supply backorder. Although it is expected that the costs can be substantially reduced in comparison with a capacitated supply environment in which the unfilled part of the order is lost rather than backlogged, we are also interested in whether full supply backordering can be counterproductive in specific situations. For clarity, we present the set of possible decision strategies and corresponding abbreviations in Table 1.

We proceed with a review of the relevant literature on supply uncertainty models, where our setting fits within the scope of single-stage inventory models with random capacity. The way we model the supply availability is inline with the work of Ciarallo et al. (1994), Güllü et al. (1999), Khang and Fujiwara (2000) and Iida (2002), where the random supply/production capacity determines a random upper bound on the supply availability in each period. Their research is mainly focused on establishing the structure of the optimal policy. For a finite horizon stationary inventory model they show that the optimal policy is of an order-up-to type, where the optimal base-stock level is increased to account for possible, albeit uncertain, capacity shortfalls in future periods.

There have been several approaches to mitigate the supply capacity uncertainty suggested in the literature, where some have explored the benefits of an alternative supply source, or the means of decreasing the uncertainty of supply itself. Our model can also be interpreted as a dual sourcing system in which the stochastically capacitated primary supplier delivers part of the order with zero lead time, while the alternative supplier has ample capacity but is only able to deliver the remaining part of the initial order with a one period delay. Assuming deterministic lead time, several papers discuss the setting in which lead times of the two suppliers differ by a fixed number of periods (Fukuda, 1964; Veeraraghavan and Sheller-Wolf, 2008). However, they all assume infinite supply capacity or at most a fixed capacity limit on one or both suppliers. For an identical lead time situation as ours, albeit uncapacitated, Bulinskaya (1964) shows the optimality of the base-stock policy and derives its parameters. However, when there is uncertainty in the supply capacity, diversification through multiple sourcing has received very little attention. The exception to this are the papers by Dada et al. (2007) and Federgruen and Yang (2009), where they study a single period problem with multiple capacitated suppliers and develop the optimal policy to assign orders to each supplier. Our model differs from the abovementioned dual sourcing models due to the fact that the alternative replenishment directly corresponds to the unfilled part of the order placed to the primary supplier, and therefore it cannot be considered as an independent ordering decision.

A recent stream of research considers the case where the information on the availability of supply capacity for the near future is provided by the supplier. In Jakšič et al. (2011) and Atasoy et al. (2012) they show how this so called advance capacity information influences the structure of the optimal policy, which is shown to be a state-dependent base-stock policy. A general assumption in stochastically capacitated single sourcing inventory models is that the part of the order above the available supply capacity in a certain period is lost to the customer. We believe this might not hold in several situations observed in practice. However, the literature that assumes the possibility of backordering the unfilled part of the customer's order at the supplier is scarce. For a continuous review system (Moinzadeh and Lee, 1989) study the system where orders arrive in two shipments, the first shipment with only random part of the items ordered, while the rest of the items arrive in a second shipment. Assuming (Q,R) policy, they present the approximate cost function and compute its parameters. Anupindi and Akella (1993) study a dual unreliable supplier system, where they assume that the unfilled part of the order is delivered with a one period delay in their Model III. A non-zero lead time setting is assumed in Bollapragada et al. (2004), where the supplier guarantees delivery either within his quoted lead time, or at most one period later. They study the two-stage serial inventory system under the assumption that approximate installation base-stock policies are followed, and evaluate the benefits of guaranteed delivery over the system with unlimited supply backlog. This way of modeling the supply backorders corresponds to what we denote in this paper as a *Full backordering* policy.

However, the situation in which the customers whose orders were backordered at the supplier may cancel their orders is rarely considered in the literature. You and Hsieh (2007) assume a constant fraction of customers are canceling their backorders. Therefore they do not consider the cancelation of backorders as a decision variable, but as a preset system parameter, which effectively reduces the demand the supplier is facing. In this paper we include the option to either partially or fully cancel backorders on the supply side as an integral part of the optimal ordering decision policy.

The remainder of the paper is organized as follows. We present our dynamic programming model incorporating supply backordering and the cancelation option in Section 2. The structure of the optimal policy and its characteristics related to the option of canceling the replenishment of the backordered supply are derived in Section 3. In Section 4 we assess the benefits of supply backordering and the value of the cancelation option through a numerical study and we point out the relevant managerial insights.

Table 1
Labeling scheme for possible decision strategies.

Label	Strategy	Description
NB	*No backordering*	Retailer is only placing regular orders and there is no supply backordering
FB	*Full backordering*	Backordered supply is always replenished in full
PB	*Partial backordering*	Retailer decides to what extent the backordered supply should be replenished
CO	*Cancelation option*	Retailer decides whether to fully cancel the replenishment of the backordered supply or not