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Managing stochastic demand in an Inventory Routing Problem with transportation procurement

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ABSTRACT

We study an Inventory Routing Problem in which the supplier has a limited production capacity and the stochastic demand of the retailers is satisfied with procurement of transportation services. The aim is to minimize the total expected cost over a planning horizon, given by the sum of the inventory cost at the supplier, the inventory cost at the retailers, the penalty cost for stock-out at the retailers and the transportation cost. First, we show that a policy based just on the average demand can have a total expected cost infinitely worse than the one obtained by taking into account the overall probability distribution of the demand in the decision process. Therefore, we introduce a stochastic dynamic programming formulation of the problem that allows us to find an optimal policy in small size instances. Finally, we design and implement a matheuristic approach, integrating a rollout algorithm and an optimal solution of mixed-integer linear programming models, which is able to solve realistic size problem instances. Computational results allow us to provide managerial insights concerning the management of stochastic demand.

1. Introduction

In the last years, a large part of the logistic activities arising in the supply chain management were reorganized to take into account the impact of the Information Technology. The modern *Enterprise Resources Planning* (ERP) systems encouraged the integration of functions and data from various areas of the supply chain. The advantages of such integration stimulated the researchers to face new logistic challenges. In this context, a high integration level occurs, for example, in the *Vendor-Managed Inventory* (VMI) systems, where a single or multiple suppliers have to make simultaneous decisions to serve a set of retailers: (1) when to serve each retailer; (2) how much to deliver to each retailer, and (3) how to schedule the deliveries in each time period. The size of the deliveries can be defined according to different resupply policies. In general, the most useful policies are of two types: the *Order-up-to Level* (OU) policy, in which the quantity delivered to each retailer fulfills the storage capacity, and the *Maximum Level* (ML) policy, in which the supplier decides a delivery size that cannot exceed the inventory capacity at each retailer. Solving the VMI implies to find solutions of a very hard combinatorial problem, named the *Inventory Routing Problem* (IRP), in which inventory and transportation decisions are integrated with the aim of minimizing the total cost resulting from using a specific resupply policy for a set of retailers. Operative constraints on the limited storage capacity at the retailers and the truckload capacity come into play when decisions have to be made.

In this paper, the focus is on the *Stochastic Inventory Routing Problem with Transportation Procurement* (*SIRP-TP*). In this problem, decisions on when and how much to deliver to a set of retailers are made over a finite and discrete planning horizon. The supplier has a limited production capacity at each time period, the demands of the retailers are modeled by discrete random variables and deliveries are performed using transportation procurement. We assume that the size of all deliveries is defined in accordance to the OU policy. This situation arises, for example, when inventory decisions related to a set of retailers occur in a VMI system where small package deliveries, fulfilling the capacity requirements, are frequently needed over a finite time horizon. Another application can be found in supermarket delivery, fuel delivery, refilling of vending machines. In these cases, transportation procurement can greatly reduce operating costs. An application is when transportation services are bought via Third Party Logistics (3PL) marketplaces that allow to pay the lowest transportation cost. Another application is when companies outsource last-mile deliveries of

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small quantities to carriers working as a freight contractors in a regional district. This allows a consolidation of the loads that significantly reduces the overall transportation cost. Additionally, administrative effort and legal problems of the vendor companies are reduced as the drivers are employees of a given transportation company or even self-employees. Outsourced deliveries are most likely to be efficient in large areas where a few clusters of retailers need to be frequently resupplied of small volumes over a given time horizon, and where the distances for traveling between the clusters are as long as unprofitable to be covered with a private fleet. The vendor company pays a fixed price to the carrier for using its (or a part of) truckload capacity. Consequently, the prices paid by the vendor company are independent from any routing decision of the freight contractor, but they may vary period by period of the time horizon according to the different sizes of the capacity leased by the carriers. A combinatorial auction is often used to obtain the lowest price. The aim of our problem is to minimize the total expected cost, represented by the sum of different cost components: (1) the total inventory cost at the supplier, (2) the total inventory cost at the retailers, (3) the total penalty cost arising whenever a stock-out occurs at the retailers, and (4) the total transportation cost to procure the transportation capacity needed to serve the retailers.

The *SIRP-TP* can be defined as an Inventory Routing Problem, as it is a generalization of the Single Link Problem, i.e. the case with one supplier and one retailer, in which a fixed transportation cost is paid for each journey from the supplier to the retailer. The Single Link Problem is defined as an Inventory Routing Problem (see [1–3]), as there is integration of inventory and transportation management over time.

The *SIRP-TP* can be easily generalized in different ways. The extension to the case with several products is straightforward. The assumption about the transportation cost and capacity can be easily modified to obtain interesting variants of the problem, corresponding to different procurement modes of transportation services. The first is when the transportation capacity is infinite. This assumption corresponds to the case, very common in LTL, in which the sum of the maximum levels of inventory is very small with respect to the transportation capacity. The second is when several transportation capacities with different costs are made available at each time. In such a case there are several transportation companies and the supplier has to choose some of them at each time. The third is when the transportation cost function has a piecewise linear structure that alternates flat and increasing parts. This assumption corresponds to the classical case in which discount schemes are applied, i.e. the unit transportation cost decreases when the quantity increases (see for instance [4,5]).

The main scientific contributions of this paper can be summarized in the following points: (a) providing a mathematical formulation of the *SIRP-TP* with deterministic demand, (b) proving that the *SIRP-TP* is NP-hard even in the deterministic case, (c) proving that the classical benchmark policy, based just on the average demand, can be infinitely worse than the optimal policy, (d) providing a stochastic dynamic programming formulation of the problem, (e) implementing an exact dynamic programming algorithm to determine an optimal policy in small instances, (f) designing and implementing a rollout-based matheuristic algorithm to determine a near-optimal policy, (g) providing managerial insights in the management of stochastic demands.

The remainder of the paper is organized as follows. A literature review is presented in Section 2. The problem is formally described in Section 3. The deterministic version of the problem and the corresponding complexity analysis is presented in Section 4. The benchmark policy is described and analyzed in Section 5. The dynamic programming formulation of the problem is presented in Section 6. The exact dynamic programming algorithm and the rollout-based matheuristic are described in Section 7. Computational results and the corresponding managerial insights are shown in Section 8.

2. Literature review

Scientific research on Inventory Routing Problems (IRP) has sped up quickly in the last years. A large number of papers studying a few variants of the classical IRP have been produced, also inspired by real-cases. For a complete and well-organized analysis on the IRPs we refer to the recent survey of Bertazzi et al. [6], Andersson et al. [7], Bertazzi and Speranza [1], Bertazzi and Speranza [2] and Coelho et al. [3]. Real cases are studied in Kim and Kim [8], Day et al. [9] and Kopanos et al. [10].

The most relevant contributions related to the one instant time horizon stochastic IRP are represented by the works proposed by Federgruen and Zipkin [11], Golden et al. [12], Dror et al. [13], Federgruen et al. [14], Bassok and Ernst [15], Herer and Levy [16], Bard et al. [17] and Berman and Larson [18]. In case of finite time horizon and stochastic demands, the major contributions are those proposed by Bell et al. [19], Gaur and Fisher [20] and Dauzère-Pérès et al. [21]. The mathematical model used to describe our problem is fundamentally rooted in modeling IRP with a stochastic demand. In this context, we start the analysis from the pioneering paper by Minkoff [22], who first proposed to model the coordination problem between different replenishment decisions as a Markov chain, in which the state in a given time period is defined by the inventory levels of all the retailers within such period, while the actions for moving from a state to another state are represented by routing decisions. The goal is to minimize the short and long-term costs of the action taken. This approach is applied to the setting of stochastic demands and with an unlimited number of vehicles by Minkoff [22]. The same approach proposed by Minkoff [22] is applied by Minkoff [23] applied to an IRP with stochastic demands, in which a fleet with a limited number of vehicles is used and only direct deliveries are allowed. They developed a dynamic programming-based approximation method to compute the optimal value function of the IRP. Such IRP problem can be almost decomposed into as many subproblems as the retailers. Then, the solutions of these subproblems are used to estimate the expected value of long-term costs, while a greedy heuristic is developed to evaluate the expected total discounted value. In this approach stock-outs are allowed, but no backordering is permitted. In the next paper, Kleywegt et al. [24] extended the approach proposed in their above work by allowing multiple deliveries per trip. In Adelman [25,26], an IRP with stochastic demands is dealt by using the same approximated value function defined by Minkoff [22] with the meaning of marginal prices for delivering. Such optimal dual prices are used into a linear program in order to approximate the future costs of current actions. Due to the infinite planning horizon, the main ideas on the basis of the works proposed by Hvattum and Løkketangen [27] and Hvattum et al. [28] is that the stocasticity can be modeled through a finite scenario tree. In Hvattum and Løkketangen [27] a GRASP algorithm is developed for the scenario tree problem which is constructed by assuming a progressive increase in the delivery volumes. In Hvattum et al. [28] a progressive hedging algorithm is applied to the problem, in which all identical sub-paths of the scenario tree are associated with the same decisions. Recently, Yu et al. [29] addressed a stochastic IRP with split delivery. The authors introduced different service levels at the suppliers and retailers with the aim of taking into account the stock-out. Such problem is modeled as an approximate stochastic IRP exploiting the transformation of stochastic components in deterministic ones, and near-optimally solved by using an hybrid approach based on a