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An informative column generation and decomposition method for a production planning and facility location problem

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ABSTRACT

This paper develops an informative column generation and decomposition method for a capacitated multi-item lot sizing and facility location problem with backlogging. The method hybridizes column generation to achieve a relaxed linear solution and a lower bound and a decomposition method (i.e., relax-and-fix) to achieve a feasible solution. The two solutions are used for a neighborhood search procedure that fixes a subset of setup decision variables to iteratively reduce problem sizes. The relax-and-fix method is applied again to solve these smaller-size restricted problems with a purpose of progressively improving solution qualities. To show the effectiveness of the method, a number of computational tests are performed using newly-generated benchmark problems. Computational comparisons with a commercial solver (CPLEX) show that the proposed method provides competitive solution results.

1. Introduction

Production planning is a medium-term master planning problem that plays a crucial role in supply chain management for manufacturing companies. The companies have to make cost-efficient production plans, as failure to do so may result in a significant loss of market competitiveness. As such, practitioners and researchers have formulated the problem into mathematical models. However, the models are complicated due to the combinatorial nature of their solution space. A simple model of the production planning problem, the single-item capacitated lot sizing problem, has even been shown to be $\mathcal{NP}$-hard (Pochet and Wolsey, 2006). The model complexities have motivated an amount of sustained research in the field (Drexl and Kimms, 1997; Karimia et al., 2003; Pochet and Wolsey, 2006; Jans and Degraeve, 2007, 2008).

Lot sizing is a major component of production planning that determines lot sizes for producing various items under limited resources over a finite time horizon. The problem has been widely studied in the literature. The early research was concentrated on single-level lot sizing problems. Zangwill (1966) and Federgruen and Tzur (1993) proposed dynamic programming approaches; Pochet and Wolsey (1988) presented strong valid inequalities and a facility location formulation for which its linear programming (LP) relaxation is proved to give an optimal solution to the single-item uncapacitated problem; Millar and Yang (1994) proposed two Lagrangian relaxation-based methods; Song and Chan (2005) proposed a method for the problem on a single machine with a finite production rate; and Kucukyavuz and Pochet (2009) defined the convex hull for the problem with backlogging. The later research trend was more switched to multi-level lot sizing problems. Akartunal and Miller (2009) proposed a relax-and-fix heuristic; Wu et al. (2011) proposed a lower bound and upper bound guided method; and Toledo et al. (2013) proposed a hybrid multi-population genetic algorithm. We refer readers to Jans and Degraeve (2008) and Buschkuhl et al. (2010) for detailed reviews.

In a modern globalized manufacturing environment, multinational companies often distribute production in different regions in order to meet internationally-wide customer needs and to reduce duty and logistic costs. This motivates the needs to simultaneously make production planning and facility location decisions. Romeijn et al. (2010) introduced an integrated production planning and facility location problem that generalizes traditional lot sizing problems by taking into account facility location decisions. The mathematical model is defined over a set of facilities, customers, and time periods. The demand of customers in each period is given and must be satisfied on time. A customer is assigned to a facility to meet demand through production and inventory-holding decisions. There is a transportation cost

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associated with facilities and customers. Each facility has an operating cost if any customers are assigned to the facility. The goal is to minimize the sum of facility-operation, transportation, production, and inventory-holding costs.

This paper extends the problem by considering capacities and having allowance of backorders. Late delivery is a common scenario in industry due to uncertainties of customer demand. Backlogging is therefore considered an important scenario in planning of manufacturing resources (Wu et al., 2013). We define the problem as the capacitated facility location and production planning problem with backlogging (CFLPPB). We formulate the problem into a mixed integer programming model with facility location, inventory-holding, backlogging, and lot sizing variables and define the model as the FLILB formulation. Its detailed description is given in Section 2.1. Its goal is to minimize the sum of setup, inventory-holding, backlogging, transportation, and facility-operation costs over the entire planning horizon by making optimized facility-operation, time-phased setup, production, inventory-holding, and backlogging decisions. Similar problems have been considered in the extant literature (Sambasivan and Schmidt, 2002; Sambasivan and Yahya, 2005; Sharkey et al., 2011).

The methodological contribution of this paper is to develop an informative column generation and decomposition (ICGD) method for the CFLPPB problem. On the one hand, the first application of Dantzig–Wolfe decomposition that appears in the lot sizing literature was proposed by Manne (1958), in which lot sizing problems are decomposed by item. Jans and Degraeve (2004) proposed Dantzig–Wolfe decomposition by period and showed that the decomposition method can provide at least the same or better lower bounds than the decomposition by item. Degraeve and Jans (2007) proposed a new Dantzig–Wolfe reformulation and a branch-and-price algorithm for the capacitated multi-item lot sizing problems. On the other hand, relax-and-fix has been applied to solving lot sizing problems with great success. Stadtler (2003) proposed a relax-and-fix method, called internally rolling schedules with time windows. Federgruen and Tzur (1993) developed progressive interval heuristics. Other relax-and-fix methods have been proposed in the extant literature, such as Suerie and Stadtler (2003), Sahling et al. (2009), Kim et al. (2010), and Wu et al. (2010).

The ICGD method hybridizes a mathematical programming technique (i.e., column generation) and a decomposition method (i.e., relax-and-fix) into a single framework where the column generation method achieves a relaxed linear solution and a lower bound and the relax-and-fix method achieves a feasible solution. In the framework, the combination of these two solutions informatively guides a procedure of fixing a subset of setup decision variables so as to find high quality solutions. The reduction of problem size helps in making sophisticated guesses by searching in a promising neighborhood. The problems of reduced size are then solved by the relax-and-fix method. The ICGD method exploits column generation with relax-and-fix for two reasons: the first one is that the lower bound achieved by column generation is at least the same or better than the one achieved by the linear relaxation of the formulation. Specific to the CFLPPB problem, our computational tests identify that column generation achieves better lower bounds than the linear relaxation of the FLILB formulation. To show the effectiveness of the ICGD method, extensive computational tests are performed based on a number of generated benchmark problems. Computational comparisons with a commercial solver show that the proposed ICGD method provides competitive results.

The remainder of this paper is organized as follows. Section 2 describes the ICGD method. Section 3 presents computational results and comparisons with a commercial solver. Finally, we conclude with future research directions in Section 4.

2. The informative column generation and decomposition method for the CFLPPB problem

We study the CFLPPB problem with facilities, capacities, and dynamic demand over the planning time horizon. There are J items whose demand is given over a horizon of T periods. There are K facilities in facility set $\{1, \ldots, K\}$, within which the set of facilities capable of producing item j is defined as $\mathcal{K}_j$. Ideally, demand of an item j should be satisfied on time either through production in the current period or through carried-inventory from previous periods. Otherwise, the demand can be satisfied through production at later periods but penalized with high penalty costs. Each lot of production requires a time-independent setup operation regardless of how many units are produced, but the total amount of consumed resources cannot exceed available capacity. There is only one facility that produces item j through the entire planning horizon. To satisfy the demand, the item is first delivered to the distribution center and then distributed to customers. It is assumed that there is a single distribution center in the model. We consider the level of transportation decisions between facilities and the distribution center, but do not consider transportation decisions between the distribution center and customers. In practical applications, our model is applicable to environments where the transportation of customer demand is exogenous to the firm, assigned, for instance to a third party logistics (3PL) provider. The total costs of any plan contains five components: facility-operation, setup, inventory-holding, backlogging, and transportation costs. The objective is to find a plan that minimizes the total costs without violations of capacity, demand-satisfaction, and other related constraints.

We make the following assumptions: setup time and costs are sequence-independent; setup carryover between periods is not permitted; initial inventories for all items are zero; lead times are zero as well; production costs is linear in production output and does not vary over time so that it is dropped from the model; setup and inventory-holding costs do not vary over time either; there is no inter-facility transportation; and each item is produced by only one facility through the entire planning time horizon.

2.1. Mathematical formulation

To present the problem formulation, we define the following notation:

Sets:

- $\{1, \ldots, T\}$ The set is indexed by s and/or t, where T is the number of periods in the planning time horizon.
- $\{1, \ldots, J\}$ The set is indexed by j, where J is the number of items.
- $\{1, \ldots, K\}$ The set is indexed by k, where K is the number of facilities.
- $\mathcal{K}_j$ The set of facilities capable of producing item j, $\mathcal{K}_j \subset \{1, \ldots, K\}$.
- $\mathcal{J}_k$ The set of items that can be produced at facility k, $\mathcal{J}_k \subset \{1, \ldots, J\}$.

Parameters:

- d_{jt} Demand of item j in period t.
- sc_j^k Setup cost for producing a lot of item j at facility k.
- hc_j^k Per-period inventory-holding cost for a unit of item j at facility k.
- st_j^k Setup time required for producing a lot of item j at facility k.
- a_j^k Production time required to produce a unit of item j at facility k.