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Exploration of efficiency underestimation of CCR model: Based on medical sectors with DEA-R model

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ABSTRACT

Data Envelopment Analysis (DEA) is one of the best-known efficiency evaluation methods due to its advantages in selection of weights. Many research papers have extensively discussed the issue of weight restrictions, rather than those implied in the model itself. However, this often leads to a failure to represent the relations of certain weights, as well as underestimation of the efficiency of Decision Making Units (DMUs). When analyzing the medical sectors of Taiwan with the developed models and CCR, it is found that efficiency underestimation by efficient DMUs is more serious than that of inefficient DMUs. In addition, underestimation occurs when weights are concentrated in the same output, however, every output of referenced DMU is the same times of corresponding output of targeted DMU.

1. Introduction

Efficiency is an important topic, and Data Envelopment Analysis (DEA) is one of the most famous efficiency evaluation methods. A mathematical model is established in DEA to judge efficient frontiers, and evaluate if the Decision Making Unit (DMU) is efficient. In addition, DEA permits to propose an improved package for inefficient DMU. The concept of a non-dominated solution proposed by Pareto and an index-based efficiency representation concept proposed by Farrell provide the basis for DEA (Cooper, Seiford, & Tone, 2002). With the introduction of the concept of non-dominated solutions and indices, Charnes, Cooper, and Rhodes (1978) developed a group of optimal mathematical equations for judging the efficient frontier, and calculating efficiency, which was called DEA, and the first group of mathematical expressions was named CCR, the abbreviated name of the authors.

Many studies have focused on the analysis of weight restrictions, since selection of weight represents one of DEA's advantages (e.g. Allen, Athanassopoulos, Dyson, & Thanassoulis, 1997; Liu & Chuang, 2009; Pedraja-Chaparro, Salinas-Jimenez, & Smith, 1997; Podinovski, 2007). Tracy and Chen (2005) first proposed that weight hypothesis may lead to underestimation from additional weight restrictions. However, CCR, based on $(\sum vx)/(\sum uy)$ or $(\sum uy)/(\sum vx)$, implies inherent weight restrictions, and has never been extensively discussed. Such restrictions may lead to a failure to represent the relations of certain weights, thus, output-oriented DEA-R was developed to address such problems (Despic, Despic, & Paradi, 2007). Since unnecessary and unreasonable weight hypothesis would cause CCR to underestimate the efficiency of a DMU, an input-oriented DEA-R model was developed. Another research pointed out that, this hypothesis not only underestimated efficiency, but also resulted in false low efficiency solutions (an efficient DMU was judged as an inefficient DMU). Therefore, this paper aims to further discuss the underestimation issues of an efficient DMU, and provide a deeper understanding of the instance when underestimation occurs. Andersen and Petersen (1993) and Seiford and Zhu (2003) developed a super-efficient model and a dependent model, respectively, to discuss efficient and inefficient DMUs. As both the super-efficient and dependent models were developed based on CCR, the problem of efficiency underestimations occur. Thus, this research intends to develop a pro-rated super-efficient evaluation model in an attempt to study how the efficient DMU was underestimated, and the instance when underestimation occurred.

The remainder of this paper is organized as follows: Section 1 describes the issues of efficiency underestimation, as well as two subjects that have not been discussed, which are underestimation of an efficient DMU, and when exactly does the instance of underestimation occur. Regarding the underestimation of an efficient DMU, this section discusses two high-efficiency models, with/without weight restrictions. Section 2 reviews the super-efficient model based on CCR, and proposes a super-efficient model based on DEA-R (excluding weight restrictions). Taking medical centers in Taiwan as an example, Section 3 compares the efficiency and optimal weights of CCR and DEA-R-based super-efficient models,

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and gains insight into the underestimation issues of an efficient DMU, as well as possible underestimation instances. The time point of underestimation is further discussed in Section 4. Finally, results and discussions are presented in Section 5.

2. Mathematical model

This research attempts to develop a new evaluation model according to a super-efficient concept proposed by Andersen and Petersen (1993), which could distinguish the advantages/disadvantages of both efficient and inefficient DMUs. In evaluating low efficiency, this development model differs little from previous models. In evaluating high efficiency, this development model evaluates the targeted DMUs evolution from high efficiency to low efficiency. First, the high-efficiency evaluation model based on CCR, an input-oriented high-efficiency model (Super-CCR-I), developed by Andersen and Petersen (1993), is described as follows:

$$\max \quad \bar{\theta}_o = \sum_{r=1}^{s} u_r \times y_{ro} \tag{1}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} v_i \times x_{ij} \geqslant \sum_{r=1}^{s} u_r \times y_{rj}, \quad j = 1, \ldots, n, \ j \neq o \tag{2}$$

$$\sum_{i=1}^{m} v_i \times x_{io} = 1 \tag{3}$$

$$v_i, u_r \geqslant \varepsilon > 0 \tag{4}$$

According to previous research, CCR may lead to underestimations due to excessive weight restrictions, which is inherit in the high-efficiency model based on CCR. Hence, a DEA-R-based high-efficiency model is proposed in this paper. In the next chapter, two high-efficiency models are compared to discuss the underestimation of an efficient DMU. The following is a DEA-R-based input-oriented high-efficiency model, i.e. Super-DEA-R-I:

$$\max \quad \theta_o \tag{5}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} \sum_{r=1}^{s} W_{ir} \frac{(X_{ij}/Y_{rj})}{(X_{io}/Y_{ro})} \geqslant \theta_o, \quad j = 1, \ldots, n \ j \neq o \tag{6}$$

$$\sum_{i=1}^{m} \sum_{r=1}^{s} W_{ir} = 1 \tag{7}$$

$$W_{ir} \geqslant 0, \ \ \theta_o \geqslant 0 \tag{8}$$

3. Case study and comparison of efficiency

3.1. Case study

To evaluate the possible underestimation of an efficient DMU with CCR, this research evaluates one case, and compares the results using both Super-CCR-I and super-DEA-R-I. Medical centers in Taiwan (the highest level of medical institutions in Taiwan) 2005, were selected for case study (Table 1). Many hospitals have been upgraded to medical centers through accreditation in order to receive increased budgets and payment for medical research. However, this surge of medical centers cannot concentrate their resources to support key research, thus, the advantages/disadvantages of an efficient DMU should be evaluated as an accurate evaluation by an efficient DMU could assist the Bureau of National Health Insurance in controlling the outlay of various medical services, and prevent such excessive outlay from crippling the entire health insurance system. Moreover, upon evaluation of the medical system by DEA, the outputs, such as efficiency, weights, and improvement packages, can provide reasonable explanations of practical applications. For instance, Chen, Hwang, and Shao (2005) and Katharaki (2008) evaluated the medical system by DEA.

This research selected all medical centers (21) as evaluation subjects, including seven public hospitals (33%) and private hospitals (67%). Two inputs and three outputs were selected, of which the total inputs and outputs were less than half of all DMUs in conformity with empirical rules. The inputs include: sickbeds and physicians, outputs include: out-patients, in-patients, and surgeries. Take DMU 4 for example, it serviced 2,596,143 out-patients, and 855,467 in-patients, and conducted 75,348 surgeries in 2005, with 2902 sickbeds and 973 physicians. The relevant coefficients of inputs and outputs are listed in Table 2, wherein the coefficient is no less than 0.7. There are no problems in selection of variables according to empirical rule.

3.2. Comparison of efficiency between Super-CCR and Super-DEA-R

First, efficiency between Super-CCR and Super-DEA-R models is compared. An input-oriented model was used in this research since a global budget payment system was adopted in Taiwan. The software for efficiency calculation was Excel. The efficiencies of CCR, DEA-R, Super-CCR, and super-DEA-R are listed in Table 3. If the efficiency of DMU is larger than 1, it represents that this DMU is efficient, and thus, inputs can be increased and the DMU maintains efficiency. The available input increment is equal to previous inputs multiplied by efficiency, namely, a higher efficiency means inputs can be increased, while maintaining an efficient state. When evaluating DMU 8 with Super-CCR, the evaluated efficiency is

Table 2
Correlation of input and output variables.

	I-1	I-2	O-1	O-2	O-3
I-1	1.000	0.956	0.774	0.990	0.828
I-2	0.956	1.000	0.775	0.945	0.781
O-1	0.774	0.775	1.000	0.769	0.719
O-2	0.990	0.945	0.769	1.000	0.863
O-3	0.828	0.781	0.719	0.863	1.000

Table 1
The input and output variables of Taiwan medical centers in 2005.

DMU	Sickbed	Physician	Out-patient	In-patient	Surgeries	DMU	Sickbed	Physician	Out-patient	In-patient	Surgeries
01	2618	1106	2,029,864	680,136	38,714	11	920	316	334,090	268,723	15,130
02	1212	473	1,003,707	297,719	18,575	12	3236	1023	1,954,775	920,215	56,167
03	1721	531	1,592,960	408,556	36,658	13	495	130	332,741	136,351	23,423
04	2902	973	2,596,143	855,467	75,348	14	1759	491	1,465,374	430,407	35,599
05	1389	447	1,116,161	337,523	23,803	15	1357	390	1,277,752	368,174	36,006
06	1500	547	1,476,282	378,658	22,503	16	2468	675	1,825,332	668,467	32,275
07	340	145	1,300,016	55,003	5,614	17	962	316	550,700	247,961	15,618
08	571	305	1,052,992	199,780	26,026	18	745	272	1,277,899	217,371	11,671
09	1168	369	1,849,711	326,109	30,967	19	1662	590	1,916,888	418,205	21,551
10	921	372	1,089,975	209,323	23,847	20	898	275	698,945	209,134	11,748
						21	1708	537	1,702,676	470,437	32,218