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Single machine past-sequence-dependent delivery times scheduling with general position-dependent and time-dependent learning effects

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ABSTRACT

This paper studies the single machine past-sequence-dependent (p-s-d) delivery times scheduling with general position-dependent and time-dependent learning effects. By the general position-dependent and time-dependent learning effects we mean that the actual processing time of a job is not only a function of the total normal processing times of the jobs already processed, but also a function of the job's scheduled position. We consider the following objective functions: the makespan, the total completion time, the sum of the θth ($\theta \geqslant 0$) power of job completion times, the total lateness, the total weighted completion time, the maximum lateness, the maximum tardiness and the number of tardy jobs. We show that the problems of minimization of the makespan, the total completion time, the sum of the θth ($\theta \geqslant 0$) power of job completion times and the total lateness can be solved by the smallest (normal) processing time first (SPT) rule, respectively. We also show that the total weighted completion time minimization problem, the discounted total weighted completion time minimization problem, the maximum lateness minimization problem, the maximum tardiness minimization problem and the total tardiness minimization problem can be solved in polynomial time under certain conditions.

1. Introduction

In traditional scheduling problems, most research assumes that the processing time of a job is a constant. However, in many realistic problems of operations management, both machines and workers can improve their performance by repeating the production operations. Therefore, the actual processing time of a job is shorter if it is scheduled later in a sequence. This phenomenon is known as the "learning effect" in the literature [1]. An extensive survey of different scheduling models and problems with learning effects could be found in Biskup [2], and Janiak et al. [3].

Biskup [4] considered the following model, i.e., if job J_j is scheduled in position r in a sequence, its actual processing time is

$$p_{jr}^{A} = p_j r^{a},$$

where p_j is the normal processing time of job J_j, $a \leqslant 0$ is a constant learning effect. He proved that two single machine scheduling problems can be solved in polynomial time. Janiak and Bachman [5] considered the same model with Biskup [4], and the following model, i.e., if job J_j is scheduled in position r in a sequence, its actual processing time is

$$p_{jr}^{A} = p_j - b_j r,$$

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where b_j is a learning ratio for job J_j. Kuo and Yang [6] considered the following model, i.e., the actual processing time of job J_j if it is scheduled in position r is given by:

$$p_{jr}^A = p_j\left(1+\sum_{l=1}^{r-1}p_{[l]}\right)^a,$$

where $a \leqslant 0$ is a constant learning rate and $\sum_{i=1}^{0}p_{[i]} := 0$. They proved that single machine total completion minimization problem remain polynomially solvable. Koulamas and Kyparisis [7] considered the following model, i.e., the actual processing time of job J_j if it is scheduled in position r is given by:

$$p_{jr}^A = \left(1-\frac{\sum_{l=1}^{r-1}p_{[l]}}{\sum_{l=1}^{n}p_l}\right)^a,$$

where $a \geqslant 1$ is a constant learning rate. They proved that single-machine makespan and total completion time minimization problems remain polynomially solvable. They also showed that the two-machine flowshop makespan and total completion time minimization problems remain polynomially solvable for some special cases. Cheng et al. [8] considered the following model, i.e., the actual processing time of job J_j if it is scheduled in position r is given by:

$$p_{jr}^A = \left(1-\frac{\sum_{l=1}^{r-1}p_{[l]}}{\sum_{l=1}^{n}p_l}\right)^{a_1} r^{a_2},$$

where a_1 and a_2 denote two learning indices with $a_1 \geqslant 1$ and $a_2 \leqslant 0$. They proved that some single-machine scheduling and some special cases of the m-machine flowshop problems can be solved in polynomial time. Wang et al. [9] consider a learning effect model, i.e.,

$$p_{jr}^A = p_j\left(\alpha a^{\sum_{i=1}^{r-1}p_{[i]}} + \beta\right),$$

where $\alpha \geqslant 0,\ \beta \geqslant 0$ and $0 < a \leqslant 1$ are parameters obtained empirically, and $\alpha + \beta = 1$. They proved that some single-machine scheduling problems can be solved in polynomial time. Other types of jobs with learning and/or aging effects have also been discussed; the reader is referred to papers by Janiak and Śnieżyk [10,11], Mosheiov [12], Wang et al. [13], Wang [14], Wu and Lee [15,16], Lee and Wu [17,18], Yin et al. [19], Cheng et al. [20], Wang [21,22], Huang et al. [23], Wang and Guo [24], Wang et al. [25], Wang and Li [26], Wang and Wang [27–30], Huang et al. [31], Lee [32,33], Lai and Lee [34], Yin and Wang [35], Wang and Wang [36], Wang and Wang [37], Wang et al. [38], Wang et al. [39], Wang and Wang [40], Wang et al. [41], Wang and Wang [42], Wang et al. [43], and Wang and Wang [44].

In a recent paper, Koulamas and Kyparisis [45] considered single-machine scheduling problems with past-sequence-dependent (p-s-d) job delivery times. They assumed that the p-s-d delivery time is needed to remove any waiting time-induced adverse effects on the job's condition prior to delivering it to the customer and it is therefore proportional to the job's waiting time. The objective functions were the makespan, the total completion time, the maximum lateness, and the number of tardy jobs. Their results showed these problems can be solved by simple polynomial-time algorithms. However, to the best of our knowledge, apart from the recent paper of Yang et al. [46] and Yang and Yang [47], it has not been investigated the scheduling models considering the p-s-d delivery time and learning effect at the same time. Yang et al. [46] considered single machine scheduling with (p-s-d) job delivery times and job-independent learning effect. Yang and Yang [47] considered scheduling problems with p-s-d delivery times and position-dependent processing times. For some single machine problems, they proposed the polynomial time algorithms. In this paper we extended the results of Yang et al. [46] and Yang and Yang [47] to the single machine p-s-d delivery times scheduling problem with general position-dependent and time-dependent learning effects. The phenomena of the p-s-d delivery times and learning effects can be found in real-life situations, for example, an electronic component may be exposed to certain electromagnetic and/or radioactive fields while waiting in the machine's pre-processing area and regulatory authorities require the component to be 'treated' (e.g., in a chemical solution capable of removing/neutralizing certain effects of electro-magnetic/radioactive fields) for an amount of time proportional to the job's exposure time to these fields. This treatment can be performed after the component has been processed by the machine but before it is delivered to the customer so it can be delivered with a 'guarantee'. Such a post-processing operation is usually called the job 'delivery time'. Unlike the traditional assumption of a job-specific constant delivery time in the scheduling literature, Koulamas and Kyparisis [45] further assumed that the job delivery time is proportional to the job waiting time in order to model the mandated post-processing job 'treatment', i.e., the past-sequence-dependent (p-s-d) job delivery times. On the other hand, if human interactions have a significant impact during the processing of the job, the processing time will add to the workers experience and cause learning effects.

The remaining part of this paper is organized as follows. In Section 2 we formulate the model. In Section 3 we consider several single-machine scheduling problems. In Section 4, an example is given. The last section presents the conclusions.