



# Decomposition of first-order hybrid Petri nets for hierarchical control of manufacturing systems



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## ABSTRACT

Although hybrid Petri net (HPN) is a popular formalism in modelling hybrid production systems, the HPN model of large scale systems gets substantially complicated for analysis and control due to large dimensionality of such systems. To overcome this problem, a typical approach is to decompose the net into subnets and then control the plant through hierarchical or decentralized structures. Although this concept has been widely discussed in the literature for discrete PNs, there is a lack of research for HPNs. In this paper, a new method of decomposition of first-order hybrid Petri nets (FOHPNs) is proposed first and then the hierarchical control of the subnets through a coordinator is introduced. The advantage of using the proposed approach is validated by an existing example. A sugar milling case study is analysed by using a decomposed FOHPN model and the optimization results are compared against the results of the approaches presented in other papers. Simulation results show not only an improvement in production rate, but also show the ability to control the plant online. In addition, by using the hierarchical control structure for an FOHPN model, it is possible to reduce the cost of communication links, improve the reliability of the system, maintain the plant locally, and partially redesign the system.

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## 1. Introduction

When designing a new system, one is forced to derive a mathematical model for better understanding of the behaviour of the system. For a long period of time, the processes with discrete variables and those with continuous variables have been modeled and analyzed using two completely different tools. Continuous systems are often described by differential and difference equations, transfer functions, etc., while in the discrete counterpart, state transition graphs, Petri Nets, etc., are employed.

In the last two decades, there were great interests in the processes that include both discrete and continuous parts. Hybrid systems are defined as dynamic systems that include continuous states, discrete-states and event variables [1]. In other words, as the plant has time-driven and event-driven dynamics, the controller manages both time-driven and event-driven parts. Modelling, analysis, and control of hybrid dynamical systems have been attracted great attention and a number of researches have been devoted to these topics [2]. Hybrid systems arise in many different fields, including robotics, automation, aerospace, embedded systems,

biological and chemical systems, transportation, process control, mixed-signal (analogue-digital) integrated circuits, power systems, oil operation and transportation, etc. [3–6].

A large number of approaches have been proposed to model a hybrid system [7]. Among these, hybrid Petri nets (HPNs) are very popular models [2]. These models extend ordinary Petri nets to include continuous places and transitions in order to capture the continuous dynamics of the system [2]. Since HPN is an extension of ordinary PN, it maintains PN properties such as concurrency, synchronization, mutual exclusion, and conflict resolution from ordinary PNs [8].

The hybrid Petri net model considered in this paper is referred to as first-order hybrid Petri net (FOHPN), since its continuous dynamics are modelled by first order differential equations [9]. Considering the fact that any  $n$ th order differential equation system can be transformed into  $n$  first-order differential equations with proper choice of states, one can conclude that FOHPN is general enough to model systems with practical interests. The overall hybrid net behaviour, which is a combination of both event driven and time driven dynamics, can be represented by a linear discrete time, time varying state variable model [10]. By this algebraic formalism, a manufacturing system could be described by a linear state variable model to which classical control theory may be applied.

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An FOHPN consists of continuous places holding fluid, discrete places containing a non-negative integer number of tokens, and discrete or continuous transitions. Delay times associated with discrete transitions can be either zero, deterministic, or stochastic. Firing of each continuous transition occurs with a constant speed in an interval of time and can only be changed with occurrence of an event. FOHPNs have been used in many application domains such as manufacturing systems [10,11], supply chain management [12], urban traffic [13], and fault monitoring [14].

The hybrid Petri net model of large scale systems can become very colossal. For instance, using HPNs, an intersection with a traffic light is represented by 134 places and 99 transitions in [13]. It is therefore apparent that if one tries to model all intersections of an urban traffic network with HPN, the net will become extremely large. Analysis of these large nets is complex and time consuming. To overcome this complexity, one could decompose the net into several subnets and then try to control the whole net by controlling and coordinating subnets. This solution results in a hierarchical or decentralized control structure that is discussed for discrete Petri nets [15]. For example, Nishi has proposed a Lagrangian relaxation method for decomposition of Petri nets in order to solve optimization of route planning problems for automated guided vehicles [16], and Lordache has proposed methods of decentralized supervision of Petri nets based on place invariant concept [17].

Although there are many articles regarding decomposition of discrete PNs, there is a lack of research in decomposition techniques for hybrid Petri nets. In [18], an object-oriented formalism is introduced to Petri nets associated with differential equation systems to structure the system decomposition and handle its complexity. This formalism is used for modelling of hybrid production systems; however no analysis is done using this model. In this paper, a decomposition algorithm is proposed for FOHPN which results in hierarchical control of the plant and an algorithm is proposed to calculate optimal firing speed of continuous transitions in the decomposed net.

A brief description of FOHPN is given in the next section. Section 3 is devoted to hierarchical control of systems using FOHPN modelling, and an algorithm for decomposition of FOHPN is proposed. Using the proposed algorithm, a sugar milling case study is analysed in Section 4 as an example of a mixed batch/continuous plant. The conclusion and possible future works are presented in Section 5.

## 2. First order hybrid Petri nets

A FOHPN is a structure  $N = \langle P, T, \text{Pre}, \text{Post}, D, C \rangle$  [9].  $P$  is the set of places that is partitioned into a set of discrete places  $P_d$  and a set of continuous places  $P_c$ . The set  $T = T_d \cup T_c$  is a finite set of transitions where  $T_d$  and  $T_c$  are finite sets of discrete and continuous transitions, respectively. Discrete transitions are further partitioned into a set of immediate, deterministic timed, and stochastic timed transitions. The function  $D : T_d \rightarrow \mathbb{R}^+$  specifies the timing of discrete transitions where  $\mathbb{R}^+$  represents positive real numbers. The function  $C : T_c \rightarrow \mathbb{R}_0^+ \times \mathbb{R}_\infty^+$  defines the firing speed of each continuous transition where  $\mathbb{R}_d^+$  represents  $\mathbb{R}^+ \cup \{a\}$ . For any continuous transition, we define  $C(t_i) = (V_i^-, V_i)$ , where  $V_i^-$  represents the minimum firing speed (mfs) and  $V_i$  represents the maximum firing speed (MFS) of transition  $t_i$ .

A state of FOHPN is represented by a marking of its places. A marking ( $\mathbf{m}$ ) is a function that assigns a non-negative integer number of tokens to each discrete place and a fluid volume to each continuous place. The marking of a place is influenced by firing of the transitions connected to that place. A transition could be fired if it is enabled. A discrete transition is enabled when the marking of each of its input places is greater than or equal to the weight of

the corresponding arc connecting the place and the transition. On the other hand, if the marking of all discrete input places is greater than or equal to the weight of the corresponding connecting arcs, the continuous transition is considered as enabled.

Firing of a continuous transition is equivalent to flow of tokens through the transition with a predetermined speed. The instantaneous firing speed (IFS) at time  $\tau$  of a continuous transition ( $t_j \in T_c$ ) is denoted by  $v_j(\tau)$ . If it is assumed that no discrete transition is fired at time  $\tau$  and all speeds are continuous in  $\tau$ , the marking evolution of a continuous place in time can be written as

$$\frac{dm}{dt}(\tau) = \sum_{t_j \in T_c} \mathbf{C}(p_i, t_j) v_j(\tau) \quad (1)$$

where  $\mathbf{C}$  is the incidence matrix of the net and is represented as follows

$$\mathbf{C}(p, t) = \begin{bmatrix} \mathbf{C}_{cc} & \mathbf{C}_{cd} \\ \mathbf{C}_{dc} & \mathbf{C}_{dd} \end{bmatrix} \quad (2)$$

A macro event occurs when (a) a continuous place becomes empty, or (b) a discrete transition fires, or (c) a continuous place, whose marking is increasing (decreasing), reaches a flow level that enables (disables) a set of discrete transitions. Since the first-order behaviour is considered in this paper, the IFS will be constant during the interval  $[\tau_k, \tau_{k+1}]$  where  $\tau_k$  and  $\tau_{k+1}$  are assumed to be the occurrence times of two sequential macro-events. The continuous behaviour of the net in this time interval is described as:

$$\mathbf{m}^c(\tau) = \mathbf{m}^c(\tau_k) + \mathbf{C}_{cc} \mathbf{v}(\tau_k)(\tau - \tau_k) \quad (3)$$

$$\mathbf{m}^d(\tau) = \mathbf{m}^d(\tau_k)$$

When a macro event occurs, it is possible to have a jump in the marking of discrete and continuous places. Let  $\sigma(\tau)$  be the firing count vector associated to the firing time  $\tau(k)$ , then an equation for the occurrence of a macro event can be written as follows:

$$\mathbf{m}^c(\tau) = \mathbf{m}^c(\tau^-) + \mathbf{C}_{cd} \sigma(\tau) \quad (4)$$

$$\mathbf{m}^d(\tau) = \mathbf{m}^d(\tau^-) + \mathbf{C}_{dd} \sigma(\tau)$$

Linear inequalities will be used to characterize the set of all admissible firing speed vectors. Each IFS vector in this set represents one operational mode of the system. The operator should choose the best IFS vector for the system in order to satisfy a given objective function with constraints. To compute an optimal IFS vector, an objective function must be introduced. Some possible objective functions are maximizing flows, maximizing outflows, minimizing stored flows, and minimizing the transient time. In this paper, the algorithm presented by [9] is used to compute the set of all admissible IFS vectors and select the best IFS vector for the given objective function.

## 3. Hierarchical control of FOHPN

### 3.1. Hierarchical control of large scale systems

The concept of large-scale systems can be described as complex systems composed of a number of smaller subsystems with particular functions and shared resources that are governed by interrelated goals and constraints [19]. In such systems, the typical centralized controllers may fail due to lack of a centralized information-gathering system or of centralized computing capabilities. Due to geographical distribution of components in large scale systems, the costs and the reliability of communication links cannot be neglected. On the other hand, advancement in microprocessors provides a new solution for distributed computation. Different portions of computation necessary to control the whole system is

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