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Perturbation Analysis for Optimal Production Planning Of a Manufacturing System with Subcontracting and Machine Degradation

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Abstract:

This paper deals with a manufacturing system M_1 which has to satisfy a random demand during a finite horizon given a required service level. To help meet this demand, subcontracting is used through another production system M_2 which has a random service level β. The aim of this study is to determine the production plan of the manufacturing system M_1 for each period of the horizon taking into account the machine M_1 degradation according its production rate. Baring in mind that realistically the subcontractor is not always available to satisfy each demand variation, we assume that we can only order a minimum fixed quantity -defined a priori- during the entire horizon. The optimal production plan then will correspond to the minimum sum of production (M_1 and M_2), inventory, lost sales cost and degradation cost.

Keywords: subcontraction, discrete flow model, perturbation analysis, production planning, degradation

1. INTRODUCTION

Production planning consists primarily with the adaptation of the industrial resources of the firm in order to satisfy customers demands (Feiring, 1991). This concerns the minimization of the total cost including production, inventory and lost demands which is the first action of a hierarchical decisions making process. (Buzacott and Shanthikumar, 1993) classified the related literature into two groups. The first is based on the ''fluid flow model'', where production is modeled as a continuous process with constant demand and processing times. (Akella and Kumar, 1986) showed that the optimal policy under such a model is a hedging point control if unmet demands are completely backlogged. The second group is based on the ''discrete part manufacturing system''. This group takes into account the randomness in the production times and demand arrivals, and the backlogged unmet demands (Song and Sun, 1999). Recently,(Ayed et al., 2012; Hajej et al., 2009) dealt with combined production and maintenance plans for a manufacturing system satisfying a random demand over a finite horizon. In their model, they assumed that the failure rate depends on time and on production rate. Insufficient production capacity and respect variable customer demand forced some industrial companies to call up subcontracting. (Ayed et al., 2012) used subcontracting as an independent production system in order to meet a random demand. They formulated an inventory and production problem as a constrained stochastic linear quadratic problem generalizing the HMMS model (Holt, Modigliani, Muth and Simon) (Holt et al., 1960), and they assumed that subcontractor can responds the different orders, which is not usually obvious. For realistic description of the system, a suitable model called discrete flow model (Vasic and Ruskin (2011)) (Turki et al. (2013a)) (Turki and Rezg (2014)) is used in this paper. This model is considered more realistic for discrete manufacturing systems than stochastic flow model (Markou and Panayiotou (2007)) (Turki et al. (2014)). Indeed, the discrete flow model allows tracking individual parts part by part either in performance evaluation or real-time flow control and is generally easier for simulation. Using this model, an optimization stochastic problem will be developed that seeks to minimize the total expected cost.

In this paper we propose an interesting method to optimize the system called Perturbation Analysis (PA) ((Yu and Cassandras (2004)) (Turki et al. (2013b)). This method is a technique that allows obtaining sample path derivatives of a random variable with respect to the parameters of interest (e.g., production rate, inventory level…). The relevant advantage of the Perturbation Analysis method is that the simulation based on this method allows reducing the simulation time comparing to a classical simulation method. This relevant advantage is due to the fact that the optimization algorithm based on PA determines at each step the gradient estimators of the new value of a parameter of interest. Thus, the gradient estimator value allows driving

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quickly the algorithm to the optimal value. In the literature we find Yu and Cassandras (2004) which applied the PA method on a stochastic flow model and then determined gradient estimators of throughput and buffer overflow metrics with respect to production control parameters, then the authors used them as approximate gradient estimators in simple iterative schemes for adjusting thresholds in order to optimize an function that trades off throughput and buffer overflow costs. Furthermore, the authors showed that gradient estimators are unbiased before using them in the optimization algorithm. Indeed, the main condition for making the application of PA useful in practice is to show them unbiasedness. Thereafter, these estimators could be used in stochastic approximation algorithm. Besides, to show the unbiasedness of the estimators in the discrete setting is more complicate than in the continuous setting (stochastic fluid model). However, despite this difficulty, we will use the discrete flow model to our system due to the fact that this model is more realistic and precise than stochastic fluid model that sometime does not maintain the identity of some important parameters of manufacturing systems such as service level.

The paper is organized as follows. We present the discrete flow model with the formulation in section 2. In section 3 the perturbation method is applied on the discrete flow model. Optimization algorithm and a numerical example are presented in section 4. Finally, the last section concludes the paper and gives some perspectives to our work.

2. EXPLANATION OF THE PROBLEM

In this paper, we study a randomly failing manufacturing system M_1 producing one type of product which fulfils a selling inventory *S*. The customer demand is supposed random over a finite horizon and is satisfied from the selling inventory. We assume that the maximum production rate is lower than the average demand. However, another production system is available to act as a subcontractor and to help meet the demand (figure 1). This system called subcontractor's manufacturing system M_2 is characterized by its constant production rate, its unit production cost, and its random availability rate following uniform low. The unsatisfied demands are lost and induce a certain cost. The failure rate of M_1 is increasing with both time and production rate.

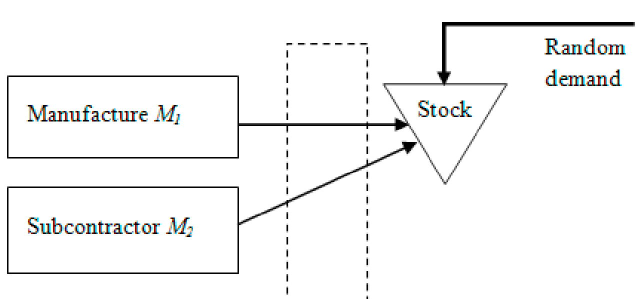

Fig. 1. Studied manufacturing system.

The following parameters are used in the model formulation:

Δt: period length of production
H : number of production periods in the planning horizon
$H.\Delta t$: finite production horizon
$u_1(k)$: production rate of the machine M_1 during period k (k=0,1,…, H-1)
u_2: constant production rate of the machine M_2 during horizon H
$\beta(k)$: machine M_2 availability rate during period k
$\hat{d}(k)$: average demand during period k (k=0, 1,…, H)
$V_d(k)$: variance of demand during period k (k=0, 1,…, H)
$s(k)$: selling inventory level at the end of period k (k=0, 1,…,H)
$\hat{s}(k)$: average selling inventory level during period k (k=0, 1,…,H)
$s_0(k)$: initial selling inventory level
$L(k)$: number of unsatisfied demands at the end of period k
cp_1: unit production cost of machine M_1
cp_2: unit production cost of machine M_2
cs: holding cost of product unit during one period.
cs^- : unit lost sales cost.
U_{max}: maximal production rate of machine M_1
U_{min} : minimal production rate of machine M_1
α: probability index related to customer satisfaction and expressing the service level.
$u_\alpha(k)$: minimum cumulative production quantity during the period k.
c_λ: unit degradation cost (for the machine M_1)
$\lambda(k,t)$: machine failure rate function during period k *(k=0, 1,…, H)*

The selling inventory level at the period $k+1$ equals to the inventory level at the period k plus the quantity produced by the machine M_1 and M_2 during period k, minus the customer demand during period k. Therefore, the selling inventory level at the period $k+1$is given by the following equation:

$$s(k+1) = s(k) + u_1(k) + \beta(k).u_2 - d(k) \tag{1}$$

The service level requirement constraint for each period is expressed by the following constraint:

$$PROB\ (s(k+1) \geq 0) \geq \alpha \tag{2}$$

The following constraint defines an upper and lower bounds on the production level of the machine M_1during each period k :

$$U_{min} \leq u_1(k) \leq U_{max} \tag{3}$$

The service level constraint is transformed into a new equivalent constraint. This new form of deterministic inequality defined a minimal cumulative production amount depending on service level requirements.

Lemma 1: