Applied Mathematical Modelling 37 (2013) 6695–6702

Contents lists available at SciVerse ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apm

Some single-machine scheduling with sum-of-processing-time-based and job-position-based processing times

Yuan-Yuan Lu [a,*], Ji-Bo Wang [b]

[a] College of Mathematics, Jilin Normal University, Siping, Jilin 136000, China
[b] School of Science, Shenyang Aerospace University, Shenyang 110136, China

ARTICLE INFO

Article history:
Received 7 February 2010
Received in revised form 26 December 2012
Accepted 9 January 2013
Available online 31 January 2013

Keywords:
Scheduling
Single-machine
Learning effect
Deteriorating jobs
V-shaped schedule

ABSTRACT

The single-machine scheduling problems with position and sum-of-processing-time based processing times are considered. The actual processing time of a job is defined by function of its scheduled position and total normal processing time of jobs in front of it in the sequence. We provide optimal solutions in polynomial time for some special cases of the makespan minimization and the total completion time minimization. We also show that an optimal schedule to be a V-shaped schedule in terms of the normal processing times of jobs for the total completion time minimization problem and the makespan minimization problem.

1. Introduction

Scheduling problems have been extensively considered in the recent literature under the assumption that the processing times of jobs may be subject to change due to learning effects and/or deteriorating jobs (Pinedo [1]). Extensive surveys of different scheduling models and problems involving jobs with deteriorating jobs can be found in Alidaee and Womer [2], and Cheng et al. [3]. We refer the reader to review the book Gawiejnowicz [4] for more details on scheduling problems with time-dependent processing times. More recent papers which have considered scheduling jobs with deteriorating jobs include Wang et al. [5], Liu and Tang [6], Lee and Wu [7], Janiak et al. [8], Lee et al. [9], Tang and Liu [10,11], Lai and Lee [12], Wei and Wang [13], Wang et al. [14], Cheng et al. [15], Lai et al. [16], Wei et al. [17], Wang and Wang [18], Wang et al. [19], Wang and Wang [20], Cheng et al. [21], Lai et al. [22]. In addition, Biskup [23] and Janiak and Rudek [24] presented extensive surveys of different scheduling models involving jobs with learning effects. More recent papers which have considered scheduling jobs with learning effects include Cheng et al. [25], Mosheiov [26], Mosheiov and Sarig [27], Lee and Wu [28], Eren [29], Lee [30,31], Lai and Lee [32], and Rudek [33]. More recent papers which have considered scheduling jobs with deteriorating jobs and learning effects include Cheng et al. [34], Yang and Chang [35], Cheng et al. [36], Wang et al. [37], Lee and Lai [38], Bai et al. [39], Wang et al. [40], Lee [41].

Cheng et al. [25] considered some scheduling problems with sum-of-processing-times-based and job-position-based learning effect, i.e., the actual processing time of a job dependents not only on the processing times of the jobs already processed but also on its scheduled position. They showed that the single-machine problems are polynomially solvable if

* Corresponding author.
E-mail addresses: luyuanyuan_2006@sina.com (Y.-Y. Lu), wangjibo75@yahoo.com.cn (J.-B. Wang).

0307-904X/$ - see front matter © 2013 Elsevier Inc. All rights reserved.
http://dx.doi.org/10.1016/j.apm.2013.01.004

the performance criterion is makespan, total completion time, total weighted completion time, or maximum lateness. They also showed that the flow shop problems are polynomially solvable under a certain condition. In this paper we focus on the single-machine scheduling problems studied by Cheng et al. [25], but with more general parameters of deterioration and learning. The rest of this paper is organized as follows. In Section 2, we formulate the single-machine scheduling model. In Section 3, we consider several scheduling problems, and prove that an optimal schedule has a V-shaped property in terms of the normal processing times of jobs. The paper closes in Section 4 by proving some concluding remarks.

2. Problem formulation

Assume that there are n independent and non-preemptive jobs $J = \{J_1, J_2, \ldots, J_n\}$ to be processed on a single machine. All the jobs starting at time 0, and the machine can handle at most one job at a time and can not stand idle time between the jobs. Let p_j denote the normal processing time of job $J_j (j = 1, 2, \ldots, n)$ and $p_{[r]}$ denote the normal processing time of a job if scheduled in the rth position in a sequence. As in Cheng et al. [25], we assume that the actual processing time of job J_j if scheduled in position r, is given by

$$p_{j[r]} = p_j\left(1 - \frac{\sum_{l=1}^{r-1} p_{[l]}}{\sum_{l=1}^{n} p_l}\right)^{a_1} r^{a_2}, \tag{1}$$

where $a_1 \leqslant 0$ ($a_2 \geqslant 0$) is the deterioration rate and $a_1 \geqslant 0$ ($a_2 \leqslant 0$) is the learning rate, and $\sum_{i=1}^{0} p_{[i]} := 0$. A schedule is a sequence of the jobs that specifies the processing order of the jobs on the machine. Obviously, if $a_1 = 0, a_2 \leqslant 0$ the model (1) is the model of Biskup [42], Mosheiov [43,44], Mosheiov and Sidney [45] and Bachman and Janiak [46]. Obviously, if $a_1 = 0, a_2 \geqslant 0$ the model (1) is the model of Mosheiov [47]. Obviously, if $a_1 \geqslant 1, a_2 = 0$ the model is the model (1) of Koulamas and Kyparisis [48]. Under a given schedule $\pi = [J_1, J_2, \ldots, J_n]$, the completion time of job J_j is given by $C_j = C_j(\pi)$. We consider some single machine scheduling under the proposed model to minimize two performance measures, namely the makespan ($C_{\max} = \max\{C_j | j = 1, 2, \ldots, n\}$) and the total completion time ($\sum_{l=1}^{n} C_l$), respectively. In the remaining part of the paper, all the problems considered will be denoted using an extension of the three-field notation for scheduling problems introduced by Graham et al. [49].

3. Single-machine scheduling problems

First, we give some lemmas, which are useful for the following theorems.

Lemma 1. $\lambda - 1 + \delta(1 - \lambda x)^a - \delta\lambda(1 - x)^a \geqslant 0$ *if* $a \geqslant 1, \lambda \geqslant 1, 0 \leqslant \delta \leqslant 1$ *and* $x \geqslant 0$.

Proof. See the proof of Lemma 2 in Cheng et al. [25]. □

Lemma 2. $\lambda - 1 + \delta(1 - \lambda x)^a - \delta\lambda(1 - x)^a \leqslant 0$ if $0 \leqslant a \leqslant 1, \lambda \geqslant 1, \delta \geqslant 1$ and $x \geqslant 0$.

Proof. Similarly to the proof of Lemma 1. □

3.1. Case 1: $0 \leqslant a_1 \leqslant 1$ and $a_2 \geqslant 0$

For the problem $1|p_{j[r]} = p_j\left(1 - \frac{\sum_{l=1}^{r-1} p_{[l]}}{\sum_{l=1}^{n} p_l}\right)^{a_1} r^{a_2}, a_1 \geqslant 1, a_2 < 0|C_{\max}$, Cheng et al. [25, Theorem 1] proved that an optimal schedule can be obtained by the smallest processing time (SPT) first rule, i.e., sequencing the jobs in non-decreasing order of p_j. The following theorem shows that the problem $1|p_{j[r]} = p_j\left(1 - \frac{\sum_{l=1}^{r-1} p_{[l]}}{\sum_{l=1}^{n} p_l}\right)^{a_1} r^{a_2}, 0 \leqslant a_1 \leqslant 1, a_2 \geqslant 0|C_{\max}$ can be solved in polynomial time.

Theorem 1. *For the problem* $1|p_{j[r]} = p_j\left(1 - \frac{\sum_{l=1}^{r-1} p_{[l]}}{\sum_{l=1}^{n} p_l}\right)^{a_1} r^{a_2}, 0 \leqslant a_1 \leqslant 1, a_2 \geqslant 0|C_{\max}$, *an optimal schedule can be obtained by the largest processing time (LPT) first rule, i.e., sequencing the jobs in non-increasing order of* p_j.

Proof. Suppose $p_j \geqslant p_k$. Let π and π' be two job schedules where the difference between π and π' is a pairwise interchange of two adjacent jobs J_j and J_k, that is, $\pi = [S_1, J_j, J_k, S_2], \pi' = [S_1, J_k, J_j, S_2]$, where S_1 and S_2 are partial sequences. Furthermore, we assume that there are $r - 1$ jobs in S_1. Thus, J_j and J_k are the rth and the $(r+1)$th jobs, respectively, in π. Likewise, J_k and J_j are scheduled in the rth and the $(r+1)$th positions in π'. To further simplify the notation, let B denote the completion time of