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Cumulative quantity control chart for the mixture of inverse Rayleigh process ☆

Sajid Ali [a,*], Muhammad Riaz [b]

[a] Department of Decision Sciences, Bocconi University, via Roentgen 1, 20136 Milan, Italy
[b] Department of Mathematics and Statistics, King Fahd University of Petroleum and Minerals, Dhahran 31261, Saudi Arabia

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ABSTRACT

The engineering processes are made up of a number of the phenomenons working together that may lead to defects with multiple causes. In order to model such types of multiple cause defect systems we may not rely on simple probability models and hence, the need arises for mixture models. The commonly used control charts are based on simple models with the assumption that the process is working under the single cause defect system. This study proposes a control chart for the two component mixture of inverse Rayleigh distribution. The proposed chart namely IRMQC chart is based on mixture cumulative quantity using the quantity of product inspected until specified numbers of defects are observed. The single cause chart is also discussed as a special case of the proposed mixture cumulative quantity chart. The control structure of the proposed chart is designed, and its performance is evaluated in terms of some useful measures, including average run length (ARL), expected quality loss (EQL) and relative ARL (RARL). An illustrative example along a case study, is also given to highlight the practical aspects of the proposal.

1. Introduction

Statistical process control (SPC) is a collection of the powerful tools used in many applied areas for the reduction of variability inherent within the process. From the SPC tool kit, control charts are very commonly used and interesting not only for practitioners but also for academic research purpose due to their flexible structure and simple graphical presentation. There are two types of available control charts i.e. attribute and variable control charts. To monitor the fraction of nonconformities of a process, attribute control charts like p, u, np and c are the well-known charts when numbers of defects follow binomial, negative binomial and Poisson processes. However, for high quality processes, especially in the field of manufacturing of integrated circuits and other automated processes these charts have certain defects e.g. low defect rate, high false alarm and the assumption of normality, etc. Thus, in such situations, the time between events (TBE) charts is an efficient approach for process monitoring, controlling and improving the process when the event's occurrence rate is very low. TBE charts monitor the time between successive occurrences of events, which is different from monitoring the proportion of events occurring in a certain sampling interval. Here, the word "events" and "time" has the different interpretation depending on the specialty of the process. The event may refer to the occurrence of nonconforming items in the manufacturing process, failure in reliability analysis, etc. However, the word time conveys the meaning of attribute or variable data observed between consecutive events of concern.

Most of the attribute TBE charts are based on the geometric distribution (e.g. CCCC) or negative binomial distribution (e.g. CCC-r). One special variable TBE chart is the cumulative quantity control chart (CQCC). As the occurrence of the events follows a Poisson process, the time between two events has an exponential distribution, and so CQCC can also be called exponential chart. Goh (1987) and Chan, Xie, and Goh (1997) proposed the cumulative count control chart (CCCC) based on the geometric process to deal with high quality processes. Later, Chan, Dennis, Xie, and Goh (2002) extended the idea of Chan et al. (2000) and proposed the cumulative probability control charts (CPCC) to overcome the difficulties associated with CQCC for the geometric and exponential distributions. An improvement to CCCC was proposed by Cheng and Chan (2010) based on the negative binomial distribution and called it CCC-r chart. An alternative to CCCC, He, Mi, and Wu (2012) proposed a new counted number between omega event attribute control chart, abbreviated as CBΩ chart. They defined word omega as an event which denotes that one observation falls into some certain defined interval, and purpose is to monitor the number of

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* Corresponding author.
E-mail address: sajidali.qau@hotmail.com (S. Ali).

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consecutive parts between successive r omega events. He et al., 2012 proposal is not so much different from the well known CCC-r chart approach. The only difference is, its discretization of events that come from continuous data.

Due to the popularity of Poisson process (and its underlying time distribution i.e. exponential distribution) in various applied fields like monitoring the accident rate in a transportation system and the rate of occurrence of congenital malformations or the volume of paperwork between errors. Zhang, Xie, and Goh (2006) introduced an exponential control chart based on the sequential sampling scheme with the self starting features. Shamsuzzaman, Xie, Goh, and Zhang (2009) proposed a control chart which is based on several individual time-between-events charts. Zhang, Shamsuzzaman, Xie, and Goh (2011) developed an economic model for the exponential chart to monitor time between events data. Acosta-Mejia (2013) suggested two geometric charts (simple and geometric based run sum chart) with the run rules. He found that these charts could be compared favorably with the two sided geometric chart with probability limits when the fraction is very small, used for detection of deterioration or improvement in process. The probability distribution of the random shift, is modeled by assuming a Rayleigh distribution. Chang and Gan (2001) proposed a cumulative sum chart for high yield processes based on geometric, Bernoulli and binomial counts. Recently, Majeed et al. (2013) proposed a mixture cumulative count control chart (MCCCC) based on the mixture model of two geometric distributions. Their proposed chart is a generalization of the CCCC proposed by Chan, Xie, and Goh (2000). However, Majeed et al. (2013) used likelihood structure, which is more specific to censored data, although they did not use censored data in control limit construction. In this article, we are proposing a correct and simplified likelihood structure, and method of estimation.

In statistics and many other applied fields of engineering the exponential is most widely used life time distribution. However, the assumption of constant hazard is true only when the event's occurrence rate is constant, and every event is being observed at the same risk level. But, in engineering processes, this assumption is violated most of the time. Thus, one needs to consider some alternative control charts based on the modified assumption where the occurrence rate of an event is not constant i.e. may increase or decrease. In statistical literature, it is well known that a Rayleigh distribution has monotone increasing failure rate. However, the inverse Rayleigh distribution has an increasing or decreasing failure rate depending upon the $q > 1.069543/\sqrt{\theta}$ or $q < 1.069543/\sqrt{\theta}$. In physics, it is used in the study of various types of radiation, such as sound, light and signal processing. Shamsuzzaman and Wu (2012) noted that Rayleigh (or inverse Rayleigh) distribution describes a distance (or deviation) from the target and is mostly used to distinguish the positional deviation in geometric tolerance. Previously, the inverse Rayleigh distribution has been used by different scholars e.g. Soliman, Amin, and Abd-ElAziz (2010) considered the inverse Rayleigh distribution based on lower record values with Bayesian and non-Bayesian approaches while Howlader, Hossain, and Makhnin (2009) used the Bayesian approach for finding prediction bounds for Rayleigh and inverse Rayleigh lifetime models. Rosaiah and Kantam (2010) discussed the acceptance sampling plan when the life test is truncated at a pre-assigned time for inverse Rayleigh distribution. Dey (2012) obtained the Bayes estimates of inverse Rayleigh distribution using squared error and LINEX (linear exponential) loss functions while Aslam and Jun (2009) proposed group acceptance sampling plans for truncated life tests based on the inverse Rayleigh and log–logistic distributions.

Since, the finished manufactured products are based on the number of components thus; these components may fail due to different reasons depending on situations that may lead to defects with multiple causes. In order to model such types of multiple cause defect systems we may not rely on simple probability models and hence, the need arises for mixture models. The commonly used control charts are based upon simple models on the assumption that the process is working under a single cause defect system. In this paper, we are introducing a new control chart namely inverse Rayleigh mixture quantity cumulative (IRMQC) based on the mixture of inverse Rayleigh distribution. Random observations taken from a population is supposed to be characterized by one of the two distinct, unknown members of the inverse Rayleigh distribution. Classical and the Bayesian approaches are used for the analysis, and various performance measures are evaluated to check the efficiency of the proposed methodology. The rest of the paper is organized as follows: In Section 2, the inverse Rayleigh quantity cumulative (IRQC) chart is introduced while in Section 3, a new IRMQC is proposed. In Section 4, some performance criteria are evaluated for the proposed methodology. These performance criteria are: average run length, average length of inspection, extra quadratic loss and relative average run length. The Bayesian and the classical methods for the estimation of unknown parameters are also discussed in the same section. In Section 5, an illustrative example to explain how in real situations, the proposed methodology could be used is discussed with a case study. Finally, we conclude the paper in Section 6.

2. The IRQC

Let us assume that q be the quantity of product inspected to observe a defect which follows an inverse Rayleigh distribution (IRD) with expected value $E(q) = \sqrt{\theta\pi}$ and cumulative distribution function (CDF)

$$F(q, \theta) = \exp\left(-\theta/q^2\right), q > 0. \quad (1)$$

where θ is the scale parameter of the model. A two sided IRQC can be constructed by putting Eq. (1) equal to $\alpha/2, 1-\alpha/2$ and $1/2$ which results in the lower control limit (LCL), upper control limit (UCL) and central limit (CL) respectively (cf. Chan et al., 2000) where the probability of false alarm rate is α (was prespecified). The simplified form of control limits can express as follows: UCL: $q_U = \sqrt{\frac{-\theta_0}{\ln(1-\alpha/2)}}$, CL: $q_C = \sqrt{\frac{\theta_0}{\ln(2)}}$ and LCL: $q_L = \sqrt{\frac{-\theta_0}{\ln(\alpha/2)}}$.

For two sided IRQC, the plotting statistics q (i.e. the quantity of product inspected) is plotted against the sample number when the defect is observed. If the point is plotted below the LCL, then it is a signal that process has deteriorated while if a point is plotted above the UCL, then it is the sign of process improvement. Whenever the specified number of defects are observed, q is reset to zero. A one sided (single limit) IRQC can be obtained by equating Eq. (1) equal to α for which LCL becomes $q_L = \sqrt{\frac{-\theta_0}{\ln(\alpha)}}$ without UCL. The performance of IRQC has no question mark as contrast to the well-known charts, especially for LCL.

Remark 2.1. Asymptotic behavior of these control limits can be derived as follows: $\theta_0 \to 0, q_L = q_C = q_U \to 0$ and when $\theta_0 \to \infty$, $q_L = q_C = q_U \to \infty$.

In many practical situations, the manufactured product is not a result of a single production plant or from one company e.g. the personal computer is a combination of hundred components not furnished from one company. Hence, in such cases, there is more than one factor causing a defect in a final product. Let us suppose that the population of the defective items can be divided into two or more sub-populations such that the rate of defects produced by each sub-population is different. For example, the personal computer's components like RAM, CPU and other communication devices are produced in different countries; while to form personal computer, these components are assembled in different countries