Information Sciences 274 (2014) 303–309

Contents lists available at ScienceDirect

Information Sciences

journal homepage: www.elsevier.com/locate/ins

# Single machine scheduling with sum-of-logarithm-processing-times based deterioration

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ARTICLE INFO

*Article history:*
Received 30 June 2009
Received in revised form 4 January 2011
Accepted 3 March 2014
Available online 15 March 2014

*Keywords:*
Scheduling
Single machine
Deteriorating job

ABSTRACT

In this study we consider the single machine scheduling problems with sum-of-logarithm-processing-times based deterioration, i.e., the actual job processing time is a function of the sum of the logarithm of the processing times of the jobs already processed. We show that even with the introduction of the sum-of-logarithm-processing-times based deterioration to job processing times, single machine makespan minimization problem remain polynomially solvable. But for the total completion time minimization problem, we show that the optimal schedule is not always V-shaped with respect to job normal processing times. Heuristic algorithms and computational results are presented for the total completion time minimization problem.



## 1. Introduction

In classical scheduling problems, the processing times of jobs are assumed to be constant values. However, in many/various practical/real life settings/applications, job processing times are an increasing function of their starting times or their positions in the sequence. This phenomenon, known as deterioration, has been extensively studied in the last decade in various machine settings and performance measures. Extensive surveys of research related to scheduling deteriorating jobs can be found in Alidaee and Womer [1], and Cheng et al. [3]. We refer the reader to book Gawiejnowicz [7] for more details on scheduling problems with time-dependent processing times.

Recently, Wu et al. [20] considered single-machine total weighted completion time scheduling problem under linear deterioration. They proposed a branch-and-bound method and several heuristic algorithms to solve the problem. Toksar and Guner [17] considered the parallel machine earliness/tardiness (ET) scheduling with simultaneous effects of learning and linear deterioration, sequence-dependent setups, and a common due-date for all jobs. They introduced a mixed nonlinear integer programming formulation for the problem. Wu and Lee [19] considered single-machine group scheduling problems with deteriorating setup times and job processing times. Cheng et al. [4] considered some scheduling problems with the phenomena of job deterioration and learning exist simultaneously. They showed that the single-machine problems are polynomially solvable if the performance criterion is makespan, total completion time, total weighted completion time, or maximum lateness. Cheng et al. [5] considered some scheduling problems with the actual job processing time is a function

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http://dx.doi.org/10.1016/j.ins.2014.03.004
0020-0255/

of jobs already processed. Lee et al. [10] considered a total completion time scheduling problem in the $m$-machine permutation flow shop with deteriorating jobs. They proposed a dominance rule and an efficient lower bound to speed up the searching for the optimal solution. Lee et al. [11] considered a single machine scheduling with a new deterioration model where the actual job processing time is a function of jobs already processed. They showed that the makespan problem remains polynomially solvable under the proposed model. Tang and Liu [16] considered two scheduling problems for a two-machine flowshop where a single machine is followed by a batching machine. The first problem is that there is a transporter to carry the jobs between machines. The second problem is that there are deteriorating jobs to be processed on the single machine. For the first problem with minimizing the makespan, they formulate it as a mixed integer programming model and then proved that it is strongly NP-hard. A heuristic algorithm is proposed for solving this problem and its worst case performance is analyzed. For the second problem, they derived the optimal algorithms with polynomial time for minimizing the makespan, the total completion time and the maximum lateness, respectively. Wang et al. [18] considered single machine scheduling problem with time-dependent deterioration. They showed that, even with the introduction of time-dependent deterioration to job processing times, the single-machine makespan minimization problem remains polynomially solvable. In general, the deterioration models can be classified into three types, namely time-dependent deterioration (see, e.g., [1,3,10,12,16,17,19,20]), position-dependent deterioration (see, e.g., [2,13,14]) and sum-of-processing-times-based deterioration (see, e.g., [4,11,15,18]).

However, the actual processing time of a given job drops to very large precipitously as the number of jobs increases in the position-based deterioration model and when the normal job processing times are large in the sum-of-processing-times-based deterioration model. Motivated by this observation, we propose a new deterioration model where the actual job processing time is a function of the sum of the logarithm of the processing times of the jobs already processed. This model is adopted from Cheng et al. [6] and Wang et al. [18]. The sum-of-processing-times-based deterioration model can be described by the following example. There are some products that need to be processed by a cutting tool. Because of wear of the cutting tool, the time required for processing a single product increases with respect to the processing time of products already executed.

The remaining part of this study is organized as follows. In Section 2, we formulate the model. In Section 3, we consider several single machine scheduling problems. In Section 4, we give some heuristics and computational results for the total completion time problem. The last section presents the conclusions.

## 2. Problems description

There are given a single machine and $n$ independent and non-preemptive jobs $J = \{J_1, J_2, \ldots, J_n\}$ that are immediately available for processing. The machine can handle one job at a time and preemption is not allowed. Each job $j$ has a normal (basic) processing time $p_j$ with $\ln p_j \geqslant 1$. Cheng et al. [6] considered the following model, i.e., the actual processing time of job $J_j$ if it is scheduled in the $r$th position in a sequence is $p_{jr} = p_j\left(1 + \sum_{i=1}^{r-1} \ln p_{[i]}\right)^a$, where $p_{[i]}$ denotes the normal processing time of job scheduled in the $i$th position in the sequence, and $a \leq 0$ is the learning index. Wang et al. [17] considered the model where processing time of job $J_j$ is $p_{jr} = p_j\left(1 + \sum_{i=1}^{r-1} p_{[i]}\right)^a$, where $a \geqslant 0$ denote deterioration index. In this paper, we propose a new model stem from Cheng et al. [6] and Wang et al. [18]. Specially, the actual processing time of job $J_j$ if it is scheduled in the $r$th position in a sequence is

$$p_{jr} = p_j\left(1 + \sum_{i=1}^{r-1} \ln p_{[i]}\right)^a,$$

where $p_{[i]}$ denotes the normal processing time of job scheduled in the $i$th position in the sequence, and $a$ denote deterioration index with $0 \leqslant a \leq 1$. In the remaining part of the paper, all the problems considered will be denoted using the three-field notation schema $\alpha \mid \beta \mid \gamma$ introduced by Graham et al. [8].

Suppose that $\pi$ and $\pi'$ are two job schedules. The difference between $\pi$ and $\pi'$ is a pairwise interchange of two adjacent jobs $J_i$ and $J_j$, i.e., $\pi = [S_1, J_i, J_j, S_2]$ and $\pi' = [S_1, J_j, J_i, S_2]$, where $S_1$ and $S_2$ each denote a partial sequence. Furthermore, we assume that there are $r-1$ jobs in $S_1$. Thus, jobs $J_i$ and $J_j$ are the $r$th and $(r+1)$th job in $\pi$, whereas jobs $J_j$ and $J_i$ are scheduled in the $r$th and $(r+1)$th position in $\pi'$. In addition, let $A$ denote the completion time of the last job in $S_1$. Under $\pi$, the completion times of jobs $J_i$ and $J_j$ are respectively

$$C_i(\pi) = A + p_i\left(1 + \sum_{l=1}^{r-1} \ln p_{[l]}\right)^a \tag{1}$$

and

$$C_j(\pi) = A + p_i\left(1 + \sum_{l=1}^{r-1} \ln p_{[l]}\right)^a + p_j\left(1 + \sum_{l=1}^{r-1} \ln p_{[l]} + \ln p_i\right)^a. \tag{2}$$

Similarly, the completion times of jobs $J_j$ and $J_i$ in $\pi'$ are respectively