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Process improvement capability index with cost – A modeling method of mathematical programming

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ABSTRACT

To improve productivity and remain competitive, businesses nowadays introduce the process capability index (C_{pm}) to evaluate the quality of their products in an effort to improve them and cut down on operation costs. This is because C_{pm} can clearly reflect process loss and yield percentage (yield%) that it is widely used in the industry. When suppliers' process capability is found to be limited in terms of C_{pm}, an improvement in product quality is required and the cost of the improvement varies depending on the source of loss. Though C_{pm} is a very good index for the evaluation of process capability, it is unable to reflect suppliers' improvement costs. Thus, this paper takes a reduction in the improvement cost into consideration and proposes the process improvement capability index (C_{PIM}). The mathematical programming model is then used to assess the confidence interval of index C_{PIM} to overcome the problem of complicated estimation of index C_{PIM}. With C_{PIM}, manufacturers are able to evaluate suppliers' ability in process improvement, particularly when the suppliers' process capability is found to be limited, to effectively reduce suppliers' improvement costs, to improve the quality of products, to enhance productivity and finally to achieve the goal of sustainable operations.

1. Introduction

As far as modern businesses are concerned, quality is by no means a new concept. In their work, entitled *The Management and Control of Quality* [1], James R. Evans and William M. Lindsay related that back in 1887, William Cooper Procter, the grandson of the founder of Procter and Gamble, had already told his employees, "The first job we have is to turn out quality merchandise that consumers will buy and keep on buying. If we produce it efficiently and economically, we will earn a profit in which you will share." Procter's statement is especially crucial to managers in manufacturing and service organizations since it underscores the three keys to making substantial profits: productivity, cost, and quality, which are interrelated. Along similar lines, Deleryd [2] maintained that the main factors to being competitive are the pursuits of cost reduction, the enhancement of product quality and improvement in productivity. In this regard, many businesses nowadays bring in the process capability index and use it to assess product quality.

The process capability index itself includes product specifications, the process mean μ and the process standard deviation σ, with the three of them, the quality of products can be evaluated. When managers of quality management assess the quality of products, they should take into consideration not only the process mean μ and the process standard deviation

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σ but also the specifications since the specifications vary from one product to another. Thus, businesses need to make a comparison of the process mean (μ), the process standard deviation (σ) and the product specifications so as to objectively assess the quality of products. The foregoing explains exactly what the process capability index is. To be more specific, the process capability index C_{pm} is a function value for the value of the process distribution parameter (μ and σ) and specification limits. Given what it includes, the process capability index C_{pm} can be used by managers of quality management to evaluate product quality.

Kane [3] proposed two process capability indices, C_p and C_{pk}; however, these two indices cannot reflect process loss because they are defined based on yield%. Chan et al. [4] proposed an index C_{pm}, which is able to adequately reflect process loss. Pearn et al. [5] emphasized that it is because index C_{pm} is capable of assessing process loss that it can faithfully reflect process capability, i.e., both process capability and process loss. Apart from this, Govaerts [6] also indicated that when the value of index C_{pm} is sufficiently high, it can even reflect process yield. The relation between index C_{pm} and process yield, as Govaerts [6] put it, is $Yield\% \geqq 2\Phi\,(3C_{pm}) - 1$. Thus, index C_{pm} can reflect not only process loss but also process yield. Index C_{pm} is defined as follows:

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{d}{3\sqrt{\sigma^2 + (\mu - T)^2}} = \frac{1}{3\sqrt{\alpha^2 + \beta^2}} \tag{1}$$

here USL and LSL are the upper and lower specification limits, respectively; μ refers to the process mean; σ is the standard deviation of the process; T is the target value; and the equation $d = (USL - LSL)/2$. $\alpha = \sigma/d$ is the process accuracy index. With a small σ value, the process standard deviation σ is small too, which indicates that the process has considerable precision. That the process has sufficient accuracy is also indicated when $\beta = (\mu - T)/d$; that is, the precise index of the process is close to zero. The closer $\beta = (\mu - T)/d$ is to zero, the closer the process mean μ is to target value T, which implies that the expected process loss is low and the process is sufficiently accurate.

According to Phadke [7] and Pearn and Chen [8], there are two factors that result in process loss. The first is a loss from a deviation of the specification from the target, which explains a lack of process accuracy. The other is a loss caused by an enormous process variation. These two factors influence index C_{pm}. The smaller the loss caused by the two factors is, the higher the index value C_{pm} is. Conversely, the more substantial the loss resulting from the two factors is, the lower the index value C_{pm} is, which suggests that process capability is limited. Obviously, C_{pm} is an excellent index to measure the process capability.

The way to improve a limited process capability, reflected in a low value of index C_{pm}, depends on which factor causes the loss. For example, if a low value of index C_{pm} is caused by the process mean μ deviating from the target, manufacturers can simply offer suppliers' technical help. As Wei et al. [9] argue, suppliers can easily improve process capability simply by making adjustments to machines' parameters, and this at a low cost. But if the low value of index C_{pm} results from an enormous process variation, i.e. the process standard deviation σ, manufacturers have to spend much more money helping their suppliers purchase new machines and supervising the quality of raw materials because it is possibly caused by old machines and varying quality of the raw materials provided by the suppliers. Although C_{pm} is a good process capability index, it cannot reflect suppliers' improvement costs. Thus, this paper takes a reduction in the improvement cost into consideration and proposes a process improvement capability index (C_{PIM}). However, the estimate of index C_{PIM} is difficult. Based the advantages of modeling method of mathematical programming [10], this study uses the mathematical programming model to assess the confidence interval of index C_{PIM} to overcome the problem of complicated estimation of index C_{PIM}.

The remainder of this paper is organized as follows: Section 2 provides a description of the improvement capability index. Section 3 derives the estimate of $\hat{C}_{PIM}$. Section 4 discusses how the confidence intervals of index C_{PIM} are obtained by modeling method of mathematical programming. Finally, Section 5 presents the conclusions.

2. Improvement capability index

As noted above, C_{pm} does not reach the required quality standard, is a result of the lack of accuracy of the process and of a lack of precision of the process. Generally speaking, it involves a lower cost to improve a limited process capability caused by the former than by the latter. As the mentioned above, though C_{pm} can be used to evaluate process capability, it cannot reflect process improvement capability. The two situations in the following paragraphs are used as examples to explain this very assertion. We assume that there are two factories, A and B, which produce the same products. Assume that T is the target value; the equation $d = (USL - LSL)/2$; $T \pm d$ is the product's upper and lower specification limits; C_1 is a cost unit to improve process precision; and that C_2 is a cost unit to improve process accuracy. In the case of factory A, the index value C_{pm} is 0.9 under the assumption that the average deviation of the process from the target is $d/6$ and that the standard deviation of the process is $d/3$. Chen et al. [11] point out that there is need to improve process capability when the process capability index value C_{pm} is less than 1. Additionally, Linderman et al. [12] reported that with the definition of the six sigma, American Motorola Corporation allows the process deviation from target to be no more than $d/4$, that is, under 1.5σ. The deviation in factory A is within the six sigma; therefore, the main reason for the process incapability is the lack of precision of the process. Furthermore, the result of the analysis by the quality department shows that lack of precision of the process was likely brought about by a shortage of raw materials, which results in frequent change of source of raw materials in the market. Besides, given the different batches of material, the quality of the raw materials varies, which possibly led to the process