Expert Systems with Applications 38 (2011) 6612–6617

Contents lists available at ScienceDirect

Expert Systems with Applications

journal homepage: www.elsevier.com/locate/eswa

Scale and cost efficiency analysis of networks of processes

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ARTICLE INFO

Keywords:
Networks of processes
Intermediate flows
Network DEA
Scale efficiency
Returns to scale
Cost efficiency

ABSTRACT

In this paper a simple way of computing technical, scale, cost and allocative efficiency scores for homogeneous networks of processes is presented. The system Production Possibility Set (PPS) is formed through the composition of the PPS of the individual processes, which, in turn, are modelled in the conventional, axiomatic way using observed data. Firstly, the overall system scale and technical efficiency are computed using the relational network DEA approach. Local Returns To Scale (RTS) can also be estimated with these models. Secondly, assuming the prices of exogenous inputs are known, a minimum cost network DEA model is solved, from which cost and allocative efficiencies are derived. The proposed approach is illustrated with a two-stage problem from the literature, showing the usefulness of a more detailed problem assessment both in terms of technical and scale efficiency and RTS and in terms of cost and allocative efficiency.

1. Introduction

Data Envelopment Analysis (DEA) is a well-known non-parametric technique that has been successfully used to assess the efficiency of Decision Making Units (DMUs) in many different sectors. Most DEA applications consider the system under study as a black box which consumes inputs and produces outputs. There are, however, some DEA approaches that consider the internal structure of the DMUs. Castelli, Pesenti, and Ukovich (2010) present an interesting classification of these approaches. In particular, there are DEA approaches that model the system as a network of interrelated processes (e.g. Avkiran, 2009; Chen, 2009; Färe & Grosskopf, 1996a, 1996b, 2000; Färe, Grosskopf, & Whittaker, 2007, chap. 12; Lewis, Lock, & Sexton, 2009; Lewis & Sexton, 2004a, 2004b; Löthgren & Tambour, 1999; Prieto & Zofío, 2007; Tone & Tsutsui, 2009; Yu, 2008a, 2008b; Yu & Lin, 2008; Yu & Lee, 2009).

Special attention has been devoted, due to their simpler structure, to multistage systems (e.g. Chen, Cook, Li, & Zhu, 2009; Cook, Liang, Yang, & Zhu, 2007, chap. 11; Kao, 2009a; Kao & Hwang, 2008; Liang, Cook, & Zhu, 2008; Liang, Yang, Cook, & Zhu, 2006; Liu & Wang, 2009; Yang, Wu, Liang, Bi, & Wu, in press) and also to parallel processes systems (e.g. Kao, 2009b). The relational network DEA approach can however be applied to general networks of processes (Kao & Hwang, 2010) even under Variable Returns to Scale (VRS) (Lozano, submitted for publication).

In this paper a simple yet intuitive way of modelling the intermediate flows between the processes will allow us to formulate easy-to-interpret relational network DEA models and relate them to a system PPS that results from the composition of conventional, individual processes' PPS. Using this system PPS a network DEA model for technical efficiency assessment can be formulated. Also, assuming that the prices of the exogenous inputs are known, a minimum cost network DEA model can be solved and cost and allocative efficiency scores can be derived.

The structure of the paper is the following. In Section 2 the problem definition, the notation and the key PPS concept are presented. In Section 3, radial input-oriented DEA models are proposed to compute scale and technical efficiency scores while in Section 4 the corresponding cost minimization network DEA model is proposed. Section 5 illustrates the proposed approach using a two-stage problem from the literature. Finally, Section 6 summarizes and concludes.

2. Problem definition, notation and PPS

In this section the type of system under study and the assumptions that will be considered are presented together with the required notation. The PPS of the individual processes and of the whole system are also defined and their properties analyzed.

Consider that a sample of observed data of a set of n DMUs is available. These observations can belong to a single system observed in different time periods or to different, independent systems. In the latter case, the systems have to be structurally homogeneous (Chen, 2009), i.e. they must consist of the same types of processes with the same interrelationships among them.

Let P be the number of processes. Some authors use the term subDMUs (e.g. Chen, 2009; Lewis & Sexton, 2004a) or Decision

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doi:10.1016/j.eswa.2010.11.077

Making SubUnits (e.g. Castelli et al., 2010) to designate the processes of which a DMU is composed. Other authors have called them production units or production nodes (e.g. Färe & Grosskopf, 1996a; Löthgren & Tambour, 1999). Other authors (e.g. Avkiran, 2009; Tone & Tsutsui, 2009) consider them as divisions of a multi-division organization. In series systems, the processes are usually called stages (e.g. Chen et al., 2009; Kao & Hwang, 2008; Liang et al., 2008; Sexton & Lewis, 2003).

Let $I(p)$ the set of exogenous inputs used in process p and, for each $i \in I(p)$, let x_{ij}^p denote the observed amount of exogenous input i consumed by process p of DMU j. Let $P_I(i)$ the set of processes that consume the exogenous input i and $x_{ij} = \sum_{p \in P_I(i)} x_{ij}^p$ the total amount of exogenous input i consumed by all processes of DMU j. Similarly, let $O(p)$ the set of final outputs of process p and, for each $k \in O(p)$, let y_{kj}^p denote the observed amount of final output k produced by process p of DMU j. Let $P_O(k)$ the set of processes that produce the final output k and $y_{kj} = \sum_{p \in P_O(k)} y_{kj}^p$ the total amount of final output k produced by all processes of DMU j.

In addition to these exogenous inputs and outputs, there exist R intermediate products generated and consumed within the system. Thus, let $P^{\text{out}}(r)$ the set of processes that generate the intermediate product r and for each $p \in P^{\text{out}}(r)$ let z_{rj}^p the observed amount of intermediate product r generated by process p of DMU j. Analogously, let $P^{\text{in}}(r)$ the set of processes that consume the intermediate product r and for each $p \in P^{\text{in}}(r)$ let z_{rj}^p the observed amount of intermediate product r consumed by process p of DMU j. Let us assume that

$$\sum_{p \in P^{out}(r)} z_{rj}^p = \sum_{p \in P^{in}(r)} z_{rj}^p \quad \forall r, \forall j \tag{1}$$

which means that the intermediate products consumed by a DMU are completely generated in-house so that there is no need to acquire them externally (as it occurs with the exogenous inputs) nor to sell them (as it occurs with the final products).

The sets $P^{\text{out}}(r)$ and $P^{\text{in}}(r)$ jointly determine the structure of intermediate flows within the system, which alternatively may be expressed through sets $R^{\text{out}}(p)$ and $R^{\text{in}}(p)$ corresponding to the intermediate products produced and consumed, respectively, by a certain process p.

Following the conventional DEA approach at the individual process level we can define the PPS of process p as

$$T_p = \left\{ (x_i^p, y_k^p, z_r^p) : \begin{array}{l} \exists \lambda_j^p \in \Lambda_p \ \forall j \quad x_i^p \geqslant \sum_j \lambda_j^p x_{ij}^p \ \forall i \in I(p) \quad y_k^p \leqslant \sum_j \lambda_j^p y_{kj}^p \ \forall k \in O(p) \\ z_r^p \geqslant \sum_j \lambda_j^p z_{rj}^p \ \forall r \in R^{\text{in}}(p) \quad z_r^p \leqslant \sum_j \lambda_j^p z_{rj}^p \ \forall r \in R^{\text{out}}(p) \end{array} \right\}$$

As in conventional DEA, the set Λ_p represent the RTS assumption for process p. Thus, $\Lambda_p^{\text{CRS}} = \{\lambda_j^p : \lambda_j^p \geqslant 0 \ \forall j\}$ corresponds to Constant Returns to Scale (CRS), $\Lambda_p^{\text{NIRS}} = \{\lambda_j^p : \lambda_j^p \geqslant 0 \forall j \quad \sum_j \lambda_j^p \leqslant 1\}$ corresponds to Non-Increasing Returns to Scale (NIRS) and $\Lambda_p^{\text{VRS}} = \{\lambda_j^p : \lambda_j^p \geqslant 0 \ \forall j \quad \sum_j \lambda_j^p = 1\}$ corresponds to VRS.

It is well known that the PPS T_p can be derived axiomatically from a set of assumptions using the minimum extrapolation principle. Thus, in the case of CRS those axioms are:

A.1. Envelopment: $(x_{ij}^p, y_{kj}^p, z_{rj}^p) \in T_p \ \forall j$

A.2. Free disposability:

$$(x_i^p, y_k^p, z_r^p) \in T_p \Rightarrow (\hat{x}_i^p, \hat{y}_k^p, \hat{z}_r^p) \in T_p \quad \forall \hat{x}_i^p \geqslant x_i^p \ i \in I(p) \quad \forall \hat{y}_k^p \leqslant y_k^p \ k \in O(p)$$
$$\forall \hat{z}_r^p \geqslant z_r^p \ r \in R^{\text{in}}(p)$$
$$\forall \hat{z}_r^p \leqslant z_r^p \ r \in R^{\text{out}}(p)$$

A.3. Convexity:

$$\left.\begin{array}{l} (x_i^p, y_k^p, z_r^p) \in T_p \\ (\hat{x}_i^p, \hat{y}_k^p, \hat{z}_r^p) \in T_p \end{array}\right\} \Rightarrow (\alpha x_i^p + (1-\alpha)\hat{x}_i^p, \alpha y_k^p + (1-\alpha)\hat{y}_k^p, \alpha z_r^p + (1-\alpha)\hat{z}_r^p) \in T_p \ \forall 0 \leqslant \alpha \leqslant 1$$

A.4. Constant returns to scale:

$$(x_i^p, y_k^p, z_r^p) \in T_p \Rightarrow (\lambda x_i^p, \lambda y_k^p, \lambda z_r^p) \in T_p \ \forall \lambda \geqslant 0$$

The NIRS case substitutes A.4 by this more restrictive assumption.

A.5. Non-increasing returns to scale:

$$(x_i^p, y_k^p, z_r^p) \in T_p \Rightarrow (\lambda x_i^p, \lambda y_k^p, \lambda z_r^p) \in T_p \ \forall 0 \leqslant \lambda \leqslant 1$$

Finally, the VRS case only assumes A.1, A.2 and A.3.

Once the PPS of the individual processes (called subtechnologies by some authors, (e.g. Färe & Grosskopf, 2000; Färe et al., 2007, chap. 12; Prieto & Zofío, 2007) have been introduced, the system PPS (also called the network technology) can be defined as the composition of these subtechnologies, i.e.

$$T = \left\{ (x_i, y_k) : \begin{array}{l} \exists (x_i^p, y_k^p, z_r^p) \in T_p \ \forall p \quad x_i \geqslant \sum_{p \in P_I(i)} x_{ij}^p \ \forall i \quad y_k \leqslant \sum_{p \in P_O(k)} y_{kj}^p \ \forall k \\ \sum_{p \in P^{\text{out}}(r)} z_{rj}^p - \sum_{p \in P^{\text{in}}(r)} z_{rj}^p \geqslant 0 \ \forall r \end{array} \right\}$$

For the network technology we can state similar properties as for the subtechnologies:

NA.1. Envelopment:

$$(x_{ij}, y_{kj}) \in T \ \forall j$$

NA.2. Free disposability:

$$(x_i, y_k) \in T \Rightarrow (\hat{x}_i, \hat{y}_k) \in T \ \forall \hat{x}_i \geqslant x_i \ \forall \hat{y}_k \leqslant y_k$$

NA.3. Convexity:

$$\left.\begin{array}{l} (x_i, y_k) \in T \\ (\hat{x}_i, \hat{y}_k) \in T \end{array}\right\} \Rightarrow (\alpha x_i + (1-\alpha)\hat{x}_i, \alpha y_k + (1-\alpha)\hat{y}_k) \in T \ \forall 0 \leqslant \alpha \leqslant 1$$

NA.4. Constant returns to scale:

$$(x_i, y_k) \in T \Rightarrow (\lambda x_i, \lambda y_k) \in T \ \forall \lambda \geqslant 0$$

NA.5. Non-increasing returns to scale:

$$(x_i, y_k) \in T \Rightarrow (\lambda x_i, \lambda y_k) \in T \ \forall 0 \leqslant \lambda \leqslant 1$$

The following results are intuitive. Nevertheless, proofs are given in the Appendix A.

Proposition 1. *If properties A.1, A.2 and A.3 hold for all subtechnologies T_p, then properties NA.1, NA.2 and NA.3 hold for the network technology T.*

Proposition 2. *If property A.4 holds for all subtechnologies T_p, then property NA.4 holds for the network technology T.*

Corollary 1. *If all the processes have CRS PPS then the system PPS is also CRS.*

Proposition 3. *If property A.5 holds for all subtechnologies T_p, then property NA.5 holds for the network technology T.*

Corollary 2. *If all the processes have either CRS or NIRS PPS then the system PPS is NIRS.*