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Cross-efficiency evaluation based on ideal and anti-ideal decision making units

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ABSTRACT

Cross-efficiency evaluation is an effective approach to ranking decision making units (DMUs) that utilize multiple inputs to produce multiple outputs. Its models can usually be developed in a way that is either aggressive or benevolent to other DMUs, depending upon the decision maker (DM)'s subjective preference to the two extreme cases. This paper proposes several new data envelopment analysis (DEA) models for cross-efficiency evaluation by introducing a virtual ideal DMU (IDMU) and a virtual anti-ideal DMU (ADMU). The new DEA models determine input and output weights from the point of view of distance from IDMU or ADMU without the need to be aggressive or benevolent to any DMUs. As a result, the cross-efficiencies measured by these new DEA models are neutral and more logical. Numerical examples are provided to illustrate the potential applications of these new DEA models and their effectiveness in ranking DMUs.

1. Introduction

Data envelopment analysis (DEA) is a methodology for assessing the performances of a group of decision making units (DMUs) that utilize multiple inputs to produce multiple outputs. It measures the performances of the DMUs by maximizing the efficiency of every DMU, respectively, subject to the constraints that none of the efficiencies of the DMUs can be bigger than one. The efficiencies measured in this way are referred to as optimistic efficiencies or the best relative efficiencies. The way to measure the optimistic efficiencies of the DMUs is referred to as self-evaluation. If a DMU is self-evaluated to have an efficiency score of one, then it is said to be DEA efficient; otherwise, the DMU is said to be non-DEA efficient. DEA efficient units are usually thought to perform better than those non-DEA efficient units.

The self-evaluation assigns the most favorable set of input and output weights to each DMU to maximize its optimistic efficiency, often leading to more than one DMU being evaluated as DEA efficient and unable to be discriminated. To improve the discrimination power of DEA, peer-evaluation was suggested as a supplement to the self-evaluation. The peer-evaluation assesses the performances of the DMUs not only in terms of their optimistic efficiencies, but also their cross-efficiencies computed using the weights determined by other peer DMUs. It is thus also called cross-efficiency evaluation.

The cross-efficiency evaluation was first proposed by Sexton, Silkman, and Hogan (1986) and was later investigated by Doyle and Green (1994) and Doyle and Green (1995). It determines a unique set of input and output weights for each DMU and then calculates its efficiencies using all the sets of weights. Accordingly, each DMU will have multiple yet different efficiency scores, whose average reflects the overall performance of the DMU. Based on average cross-efficiencies, DMUs can be compared and ranked.

Due to its strong discrimination power, the cross-efficiency evaluation has found extensive applications in the DEA literature. For example, Shang and Sueyoshi (1995) used the cross-efficiency evaluation for the selection of the most efficient flexible manufacturing systems (FMS). Green, Doyle, and Cook (1996) employed the cross-efficiency evaluation for preference voting and project ranking. Baker and Talluri (1997) applied the cross-efficiency evaluation for industrial robot selection. Sun (2002) utilized the cross-efficiency evaluation for differentiation between good and bad computer numerical control (CNC) machines. Ertay and Ruan (2005) took advantage of the cross-efficiency evaluation to determine the best labor assignment in cellular manufacturing system (CMS). Lu and Lo (2007a, 2007b) examined the economic–environmental performances of 31 regions in China by taking account of various environmental factors and then integrated the cross-efficiency evaluation with cluster analysis to construct a benchmark-learning roadmap for those inefficient regions to improve their efficiencies progressively. Wu, Liang, Wu, and Yang (2008) and Wu, Liang, and Yang (2009a) made use of the cross-efficiency evaluation in conjunction with cluster analysis for Olympic ranking and benchmarking.

Theoretical research on the cross-efficiency evaluation has also been investigated. For instance, Anderson, Hollingsworth, and

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Inman (2002) found the fixed weighting nature of the cross-efficiency evaluation where there is only one input and demonstrated with a numerical example how this unseen fixed set of weights might still be unrealistic. Sun and Lu (2005) presented a cross-efficiency profiling (CEP) model which evaluates each input with respect to the outputs that consume the input, separately. Bao, Chen, and Chang (2008) provided an alternative interpretation to the cross-efficiency evaluation from the point of view of slack analysis in DEA. Wu (2009) presented a revised benevolent cross-efficiency model and used the cross-efficiencies to construct a fuzzy preference relation for DEA ranking. Liang, Wu, Cook, and Zhu (2008a) proposed three alternative secondary goals for the cross-efficiency evaluation. Liang, Wu, Cook, and Zhu (2008b) also generalized the concept of cross-efficiency to game cross-efficiency and presented a convergent iterative algorithm to derive Nash equilibrium point.

Wu, Liang, and Chen (2009) suggested a modified DEA game cross-efficiency model by appending an extra constraint to avoid producing negative cross-efficiencies under variable returns to scale (VRS) and applied it for Olympic rankings. Wu, Liang, and Yang (2009b) calculated ultimate efficiency scores by weighting cross-efficiency values rather than simply averaging them, where the weights for ultimate efficiency scores were determined by using the Shapley value in cooperative game. Wu, Liang, Yang, and Yan (2009) developed a bargaining game model for the cross-efficiency evaluation. In their bargaining game model, each DMU was seen as an independent player and the bargaining solution between the optimistic efficiency and the cross-efficiency were obtained by using the classical Nash bargaining game model. Wu, Liang, Zha, and Yang (2009) developed a mixed integer programming model for the cross-efficiency evaluation to find the best ranking order for each DMU.

Recently, Wang and Chin (2010) proposed a neutral DEA model for the cross-efficiency evaluation and extended it to cross-weight evaluation. The neutral DEA model does not require the DM to make a difficult choice between aggressive and benevolent formulations. It determines the input and output weights only from the perspective of the DMU that is under evaluation, without being aggressive or benevolent to the other DMUs. The cross-efficiencies computed in this way turn out to be neutral and more logical.

In this paper, we provide four more neutral DEA models for cross-efficiency evaluation from the perspective of multiple criteria decision analysis (MADA), which are built using ideal DMU and anti-ideal DMU. These new DEA models provide more insights into the cross-efficiency evaluation and enhance the theory and methodology of DEA. The remainder of the paper is organized as follows. Section 2 gives a brief introduction to the cross-efficiency evaluation and its main formulations. The new DEA models for cross-efficiency evaluation are developed in Section 3. Numerical examples are examined in Section 4. The paper concludes in Section 5.

2. Cross-efficiency evaluation

Consider n DMUs that are evaluated in terms of m inputs and s outputs. Let x_{ij} and y_{rj} be their input and output values for $i = 1, \ldots, m$; $r = 1, \ldots, s$ and $j = 1, \ldots, n$. The efficiencies of the n DMUs are measured by the following CCR model (Charnes, Cooper, & Rhodes, 1978):

$$\begin{aligned} \text{Maximize} \quad & \theta_{kk} = \frac{\sum_{r=1}^{s} u_{rk} y_{rk}}{\sum_{i=1}^{m} v_{ik} x_{ik}} \\ \text{Subject to} \quad & \theta_{jk} = \frac{\sum_{r=1}^{s} u_{rk} y_{rj}}{\sum_{i=1}^{m} v_{ik} x_{ij}} \le 1, \quad j = 1, \ldots, n, \\ & u_{rk} \ge 0, \quad r = 1, \ldots, s, \\ & v_{ik} \ge 0, \quad i = 1, \ldots, m, \end{aligned} \tag{1}$$

where v_{ik} ($i = 1, \ldots, m$) and u_{rk} ($r = 1, \ldots, s$) are input and output weights. The meaning of the above CCR model is to find a set of input and output weights that are most favorable to DMU_k. By using Charnes and Cooper transformation (Charnes & Cooper, 1962), model (1) can be transformed into the following linear program (LP) for solution:

$$\begin{aligned} \text{Maximize} \quad & \theta_{kk} = \sum_{r=1}^{s} u_{rk} y_{rk} \\ \text{Subject to} \quad & \sum_{i=1}^{m} v_{ik} x_{ik} = 1, \\ & \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0, \quad j = 1, \ldots, n, \\ & u_{rk} \ge 0, \quad r = 1, \ldots, s, \\ & v_{ik} \ge 0, \quad i = 1, \ldots, m. \end{aligned} \tag{2}$$

Let u_{rk}^* ($r = 1, \ldots, s$) and v_{ik}^* ($i = 1, \ldots, m$) be the optimal solution to the above LP model. Then, $\theta_{kk}^* = \sum_{r=1}^{s} u_{rk}^* y_{rk}$ is referred to as the optimistic efficiency or CCR-efficiency of DMU_k, which reflects the self-evaluation of DMU_k; whereas $\theta_{jk} = \sum_{r=1}^{s} u_{rk}^* y_{rj} / \sum_{i=1}^{m} v_{ik}^* x_{ik}$ is referred to cross-efficiency of DMU_j and reflects the peer-evaluation of DMU_k to DMU_j ($j = 1, \ldots, n$; $j \ne k$).

Model (2) is solved for each DMU, respectively. As a consequence, there will be n sets of input and output weights available for the n DMUs and each DMU will have $(n - 1)$ cross-efficiencies plus an optimistic efficiency. These efficiencies form a cross-efficiency matrix, as shown in Table 1, where θ_{kk} ($k = 1, \ldots, n$) are the optimistic efficiencies of the n DMUs, i.e. $\theta_{kk} = \theta_{kk}^*$.

Due to the fact that model (2) may have multiple optimal solutions. To resolve the non-uniqueness issue of input and output weights, Sexton et al. (1986) suggested introducing a secondary goal to optimize the input and output weights while keeping unchanged the CCR efficiency. The most widely used secondary goals were suggested by Doyle and Green (1994) and are provided below:

$$\begin{aligned} \text{Minimize} \quad & \sum_{r=1}^{s} u_{rk} \left(\sum_{j=1, j \ne k}^{n} y_{rj} \right) \\ \text{Subject to} \quad & \sum_{i=1}^{m} v_{ik} \left(\sum_{j=1, j \ne k}^{n} x_{ij} \right) = 1, \\ & \sum_{r=1}^{s} u_{rk} y_{rk} - \theta_{kk}^* \sum_{i=1}^{m} v_i x_{ik} = 0, \\ & \sum_{r=1}^{s} u_{rk} y_{rj} - \sum_{i=1}^{m} v_{ik} x_{ij} \le 0, \quad j = 1, \ldots, n, \\ & u_{rk} \ge 0, \quad r = 1, \ldots, s, \\ & v_{ik} \ge 0, \quad i = 1, \ldots, m, \end{aligned} \tag{3}$$

Table 1
Cross-efficiency matrix for n DMUs.

DMU	Target DMU				Average cross-efficiency
	1	2	...	n	
1	θ_{11}	θ_{12}	...	θ_{1n}	$\frac{1}{n}\sum_{k=1}^{n}\theta_{1k}$
2	θ_{21}	θ_{22}	...	θ_{2n}	$\frac{1}{n}\sum_{k=1}^{n}\theta_{2k}$
⋮	⋮	⋮	⋮	⋮	⋮
n	θ_{n1}	θ_{n2}	...	θ_{nn}	$\frac{1}{n}\sum_{k=1}^{n}\theta_{nk}$