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Transformation of optimization problems in revenue management, queueing system, and supply chain management

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ABSTRACT

For revenue optimization problems in the literature on revenue management, supply chain management, and queueing systems, some assumptions (such as concavity of revenue functions or increasing generalized failure rate) are often needed to ensure the problems to be analytically tractable. We show that these assumptions are not necessary. For this, we present and study a parametric revenue maximization problem to unify some problems in the literature. Without the usual assumptions, we transform the problem into an equivalent one where the revenue function is increasing, continuous and concave. We then apply the transformation method to a continuous time revenue management problem and conclude that the monotone results are robust to demand function and allowable price set. Also, we apply the transformation method to study a parametric cost minimization problem. We further apply our method to two optimal control problems in queueing systems and an inventory control problem in a supply chain with price-only contract.

1. Introduction

Revenue maximization problems appear frequently in the literature, typically those on revenue management (Gallego and van Ryzin, 1994), optimality of queueing systems (Lippman, 1975) and supply chain management (Lariviere and Porteus, 2001). To ensure these problems to be analytically tractable there often needs some assumptions. Ziya et al. (2004) summarize and discuss three famous assumptions presented in the literature. The first two are the concavity of the revenue function with demand and price, respectively, and the third is the increasing generalized failure rate (IGFR) of the demand distribution function under which the revenue function is unimodal. Ziya et al. (2004) show that none of these assumptions implies any other. These assumptions appear in papers concerning revenue management, inventory and pricing in supply chain management, network services, auction and mechanism design, and price competition (Ziya et al., 2004). However, we no longer need these assumptions, as shown in this paper.

We first present a parametric revenue maximization problem to unify several revenue maximization problems discussed in the literature. Without the assumptions presented in the literature, we transform the problem into an equivalent well structured one in which the revenue function is increasing, continuous and concave. Thus, the resulting maximization problem is analytically tractable. The transformation here is algorithmic. We illustrate the problem and the results by an optimal arrival control in queueing systems, which is not concerned in Ziya et al. (2004).

We then apply the transformation to study the continuous time revenue management. Revenue management deals with pricing and allocation problems in many industries of selling fixed stock items over a finite horizon by controlling price. These industries include airlines selling seats before planes' departing, hotels' renting rooms before midnight, and retailers' selling seasonal items with long procurement lead time. The study on revenue management dates back to Littlewood (1972) for a stochastic two-fare and single-leg problem in the airlines. Li (1988) presents a continuous time model with demand of a controlled Poisson process. Gallego and van Ryzin (1994) study continuous time revenue management, where demands (customers) arrive according to a homogeneous Poisson process with price related demand rate, and price is chosen from the set $[0,\infty)$. They assume a *regular demand function*, that is, the corresponding revenue function (i.e., the demand rate times price) is a continuous, bounded and concave function of the demand rate, and tends to zero as the demand rate tends to zero. With the regular demand function, they show monotonicity and concavity of the optimal expected revenue and monotonicity of the optimal pricing policy.

The work of Gallego and van Ryzin (1994) has been extended into several directions: (1) to relax the assumption of the regular demand, for example, in Zhao and Zheng (2000) and Wei and

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Hu (2002); (2) to extend the allowable price set to a discrete set, for example, in Chatwin (2000), Feng and Xiao (2000a, 2000b), and Feng and Gallego (2000); (3) to study the revenue management problems in network environments, for example, in Ge et al. (2010), Dai et al. (2005), Graf and Kimms (2013); (4) to study the multi-period revenue management, for example, in Talluri and van Ryzin (2004) and Du et al. (2005); and (5) to study the revenue management in competitive environments, for example, in Netessine and Shumsky (2005), Hu et al. (2010), Huang et al. (2013), and Wei et al. (2013).

We apply the transformation to study a continuous time revenue management problem along the first and second directions pointed above. With a general demand function (that may be neither decreasing nor concave) and an arbitrary allowable price set (that can be, e.g., an interval, a discrete set, or even combination of intervals and discrete points), we show that the problem can be transformed into an equivalent one, where the revenue function is continuous and concave (i.e., the corresponding demand function is regular) and increasing. Thus, directly citing the results in the literature, e.g., Gallego and van Ryzin (1994) and Wei and Hu (2002), we get the usual monotone properties of the optimal policies and the concavity of the optimal value function. Hence, these monotone properties are robust to demand function and allowable price set.

The supply chain management is also an area concerning the revenue maximization problems. Lariviere and Porteus (2001) study a simple price-only contract where the manufacturer decides a wholesale price first and then the retailer decides an order quantity based on a random demand. The problem faced by the manufacturer is complex. Under IGFR, they show that the revenue function of the manufacturer is unimodal and then an optimal solution can be obtained analytically. We re-study the problem above and show that the manufacturer's problem can be solved analytically without IGFR.

We extend the transformation method to study a parametric cost minimization problem and get an equivalent one where the cost function is increasing, continuous and convex. A typical application of the cost minimization problem is the optimal control of service rate in queueing systems. As said in Stidham (2002), the Lippman device (Lippman, 1975) opened the gates for the application of Markov decision processes theory to queueing control problems. The idea of the Lippman device is to transform the underlying Markov process into an equivalent one in which the times between transitions are exponential random variables with a constant parameter. By applying his device, Lippman (1975) studies the optimization problems in exponential queueing systems. Later, for the optimal control problem of arrivals, Helm and Waldmenn (1984) study a general framework with multi-server queues in a random environment. For the optimal control of service rate, Jo and Stidham (1983) study the optimization problems in $M/G/1$. Stidham and Weber (1989) consider the problem of controlling the service and/or arrival rates in queues, with the objectives of minimizing the total expected cost to reach state zero and average-cost minimization over an infinite horizon. They prove that an optimal policy is monotonic in the number of customers in the system. See the details in survey papers (Stidham, 1985, 2002). However, in the literature, the analytical tractability of the optimization problems is not concerned, though is very important in computing optimal policies. Applying the transformation method, we solve the analytical tractability for an optimal service rate control in queueing systems.

The rest of the paper is organized as follows. In Section 2, we present the model of a parametric revenue maximization problem and transform it into an equivalent well structured one with a regular revenue function. Then in Section 3, we apply the transformation to study a continuous time revenue management problem without assumptions on the demand function. In Section 4, we apply the transformation method to re-study a supply chain with price-only contract. In Section 5, we generalize the transformation method to study a cost minimization problem and apply it to an optimization problem in queueing systems. Section 6 is a concluding section.

2. Parametric revenue maximization problem

In this section, we first present the model of the parametric revenue maximization problem. Without the usual conditions presented in the literature, we transform it into an equivalent one where the revenue function is increasing, continuous and concave.

2.1. Model

The model is based on a fairly standard price–demand formulation for a product (or service). There is a known mathematical relationship between price and demand. We let x denote price and y denote demand (demand in one time period, or per unit of time).

In the model, there is a parameter t from a nonempty set $\mathcal{T}$. t may represent status for decision epoch. We require no structure for $\mathcal{T}$ and so t may be multiple representing parameters. Suppose that for each t, price x is chosen from a nonempty set P and correspondingly a nonnegative demand $d(x)$ is received (called the demand function). It is initially assumed that P is a bounded set. This assumption will be relaxed in Remark 4 below. Then, a revenue $d(x)x$ is received. Furthermore, there is a cost $d(x)\lambda(t)$ after realizing the demand $d(x)$ at t. Here, $\lambda(t)$ is nonnegative and can be interpreted as unit opportunity cost for choosing x at t. Hence, we get a profit $d(x)x-d(x)\lambda(t)$ (called as revenue function) if x is choosing at t. We thus study the following parametric maximization problem:

$$\sup_{x \in P}\{d(x)x-d(x)\lambda(t)\}, \quad t \in \mathcal{T}. \tag{1}$$

Note that this is, in fact, a family of maximization problems. We will give an example of $\lambda(t)$ in revenue management later. We want to get an optimal solution x_t^* for problem (1) for each $t \in \mathcal{T}$. For convenience, we say that x_t^* is optimal for problem $(1)_t$, or simply optimal for problem (1) when no confusion is induced.

2.2. Transformation

We study the maximization problem (1) according to the following steps. First, we reduce the price set P such that $d(x)$ is a one-to-one correspondence between price x and demand y: $y=d(x)$ and $x=p(y)$ for some function $p(y)$. So, we can transform the decision variable from price x into demand $y=d(x)$. Then, we reduce the domain of the revenue function $r(y):=yp(y)$ such that it is increasing. Finally, we revise $r(y)$ to be concave.

Step I: *An equivalent one with demand variable*. Denote by $\Lambda \equiv \{d(x)|x \in P\}$ the set of demands that are allowable under some price in P. For any given demand $y \in \Lambda$, denote by $P(y) \equiv \{x \in P|d(x)=y\}$ the set of prices that yield demand y. Surely, the set $P(y)$ may include multiple prices. But it may suffice to consider the largest one $p(y):=\sup P(y)$. The following lemma says that $p(y)$ is enough for the maximization problem (1) in the set $P(y)$. We denote by $b(x,t)=d(x)x-d(x)\lambda(t)$ for convenience.

Lemma 1. *For any given $y \in \Lambda$, suppose $p(y) \in P(y)$. Then, $b(p(y),t) \ge b(x,t)$ for all $x \in P(y)$ and $t \in \mathcal{T}$, where the equality holds if and only if $y=0$ or $x=p(y)$.*