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Solving linear design problems using a linear-fractional value function

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ABSTRACT

Previous papers developed a method to easily elicit a decision maker's (DM) preferences and account for changes in the DM's preference structure. Those preferences are modeled by piecewise linear indifference curves with varying slopes producing a piecewise linear-fractional value function. Compared with traditional optimization problems which traditionally use cost minimization or revenue maximization, this model is DM-specific, it generates a knowledge set (KS) and allows the DM to find an optimal solution based on his/her expertise and preferences. When combined with real world constraints, maximizing the DM's preferences generates a decision support system (DSS) for solving specific organizational problems. This paper develops an efficient algorithm to solve a mathematical programming problem with a linear fractional objective function that models changing DM preferences and linear constraints. A DSS is developed and its algorithm is illustrated by constructing a specific example of the DSS for scheduling a police force when the objective is to maximize the police chief's expertise and preferences regarding law enforcement.

1. Introduction

In organizational decision problems firms have to assess their core objectives and the business processes needed to fulfill them [11]. When a decision maker (DM) is trying to solve a traditional problem such as profit maximization or cost minimization in products or services mix setting, the resulting model is often a linear decision problem and the solution is based on the weight of each attribute in the objective function which is determined by profit or cost parameters. Since most of these parameters are generally deterministic, the objective function is linear with constant rates of substitution. The solution is achieved by maximizing a linear profit function or minimizing a linear cost function while taking into account the relevant constraints. In this context, DM individual preferences, or other organizational preferences, are not necessarily included in the objective function although it may be beneficial to include them in the formulations of the problem. Moreover, there are even problems in which the objective function is not clearly defined because the problem is not strictly a problem of profit maximization or cost minimization. Those issues are highlighted by Bhatt and Zaveri [2]. They argue that decision support models should assist organizations in coping with specific challenges using their own specific knowledge and competencies. The purpose of this paper is to offer a model for solving problems when the objective function is based on the DM's knowledge and preferences when these preferences may change in the domain of the problem.

Two previous papers [3,4] introduced an approach for decision making under certainty when the attribute weights are local (rates of substitution vary over the attribute space) and the indifference curves are linear. This approach is named the linear fractional (LF) model and it consists of a preference elicitation technique called the constrained choice table (CCT), the construction of a piecewise linear-fractional value function based on the decision maker's (DM) preferences as shown by his/her data in the CCT, the correction of any inconsistencies if they exist in the DM's preferences, and then assigning a value to each competing alternative by using the piecewise LF value function. This approach provides a decision support system (DSS) in which the DM can provide his/her preferences as the objective. This procedure bypasses the traditional definitions of problems as profit maximization or cost minimization. It allows the DM to translate his/her knowledge set (KS) and expertise into a set of preferences which become the objective of the DSS and the organizational setting will impose a set of constraints which will be considered when the problem is solved in order to maximize the DM's preferences.

Greasley [8] shows the importance of focusing on DM preferences by presenting a case report of a process improvement within a human resource division at a United Kingdom police force. The case report highlights the importance of constructing a DSS which focuses on the DM preferences to link an organizational reengineering effort to strategic priorities derived from a range of stakeholder interests, rather than being budget driven through a range of financial indicators. Greasley demonstrates how a DSS can be developed in a manner which business processes are evaluated based on current performance and overall importance with respect to strategic objectives.

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This paper formulates design problems that use the DM's preferences which are modeled as a piecewise LF value function. Section 2 demonstrates how the LF value function is different than the traditional profit maximization or cost minimization function (with the solution algorithm in the Section Appendix A). Section 3 provides an example for using such a value function as the objective function in a multiattribute decision problem with an infinite number of alternatives. This formulation, which is based on the DM's specific knowledge and preferences, generates a model which is specific to the DM and the organizational problem under consideration. As mentioned before, this formulation is useful in cases where the objective cannot be defined as a generic cost minimization or income maximization problem. The example in Section 3 focuses on the objective of a police chief to provide safety and security to the community. In this case, applying a generic model of cost minimization may indeed save money, but it may not provide a good solution for public safety. Even maximization of labor hours may not incorporate all the needed considerations for providing safety and security. Section 3 provides a specific DSS which is based on a model that takes into consideration the police chief's expertise, knowledge and preferences in the objective function. This DSS generates a solution that contributes to public safety. Cost control and other issues will be introduced to the model via the constraint set. This model can be the foundation of a scheduling DSS which takes into account the specific organizational setting of the problem. Section 4 concludes with summary and discussion.

2. The linear-fractional objective function

Two previous papers [3,4] developed an approach to measure a DM's preference structure with linear indifference curves where the DM's preferences and thus the rate of substitution varies over the attribute space. This approach divided the attribute space into $J+1$ regions where the slope of the indifference curve in each region can vary. Because the rate of substitution can vary within a region, the form of the value function in each region is linear-fractional (a linear function divided by a linear function). If the slope of the indifference curves within a region does not vary, then the form of the preference function reduces to the traditional additive linear function. The general form of a linear-fractional value function with $J+1$ regions (piecewise segments) is shown in Eq. (1) which emphasizes the fact that regions 1 and $J+1$ have linear value functions and regions 2 through J have linear-fractional value functions. Examples of piecewise linear-fractional value functions are plotted in [3,4] and the value function for the example in Section 3.1 is plotted in Fig. 1 with one of the hinges labeled.

$$v(x)\begin{cases} v_1(x) = c_{1,1}x_1 + c_{2,1}x_2 + \cdots + c_{n,1}x_n + \sigma_1 & \textit{if}\ \ V_0 \le v(x) \le V_1 \\ v_2(x) = \dfrac{c_{1,2}x_1 + c_{2,2}x_2 + \cdots + c_{n,2}x_n + \alpha_2}{d_{1,2}x_1 + d_{2,2}x_2 + \cdots + d_{n,2}x_n + \beta_2} & \textit{if}\ \ V_1 \le v(x) \le V_2 \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots & \vdots \\ v_J(x) = \dfrac{c_{1,J}x_1 + c_{2,J}x_2 + \cdots + c_{n,J}x_n + \alpha_J}{d_{1,J}x_1 + d_{2,J}x_2 \cdots + d_{n,J}x_n + \beta_J} & \textit{if}\ \ V_{J-1} \le v(x) \le V_J \\ v_{J+1}(x) = c_{1,J+1}x_1 + c_{2,J+1}x_2 + \cdots c_{n,J+1}x_n + \sigma_1 & \textit{if}\ \ v(x) \ge V_J \end{cases} \tag{1}$$

For an application, the specific form of the linear-fractional value function is determined by the DM completing a CCT. Based on the CCT information, the corresponding value function can be constructed. Using the formulae in Brown and Israeli [3] for the two attribute case ([4] contains the formulae for the n attribute case), the axis intercepts for each region j are $I_{1,j} = [(x_{1,j}^b)^2 + (x_{2,j}^b)^2]/x_{1,j}^b$ and $I_{2,j} + [(x_{1,j}^b)^2 + (x_{2,j}^b)^2]/x_{2,j}^b$ where superscript b means that $x_{1,j}^b$ and $x_{2,j}^b$ are the values in the CCT given by the DM (see Table 1). The hinge coordinates for each region j are $h_{1,j} = I_{1,j-1}I_{1,j}(I_{2,j} - I_{2,j-1})/(I_{1,j-1}I_{2,j} - I_{2,j-1}I_{1,j})$ and $h_{2,j} = I_{2,j-1}I_{2,j}(I_{1,j} - I_{1,j-1})/(I_{2,j-1}I_{1,j} - I_{1,j-1}I_{2,j})$. For each region j, the standardized metric is $N_j = x_j^M V_{j-1} - x_{j-1}^M V_j$ and the intermediate value is $N_j = x_j^M V_{j-1} - x_{j-1}^M V_j$. Then for region j, the linear-fractional value functions in Eq. (1) can be constructed by calculating $c_{1,j} = -N_j + h_{2,j}\mathrm{V_{j\text{-}1}} - h_{2,j}\mathrm{V_j}$, $c_{2,j} = N_j + h_{1,j}\mathrm{V_j} - h_{1,j}\mathrm{V_{j\text{-}1}}$, $\propto_j = N_j(\mathrm{h}_{1,j} - h_{2,j})$, $d_{1,j} = -x_j^M + x_{j-1}^M$, $d_{2,j} = x_j^M - x_{j-1}^M$, and $\beta_j = (x_j^M - x_{j-1}^M)(h_{1,j} - h_{2,j})$. The value $v_j(x)$ for any alternative x in region j goes from $V_{J\text{-}1}$ to V_J so that $V_{J\text{-}1} \le v_j(x) \le V_J$.

Using the linear-fractional preference function as an objective function to maximize the DM's preferences combined with linear

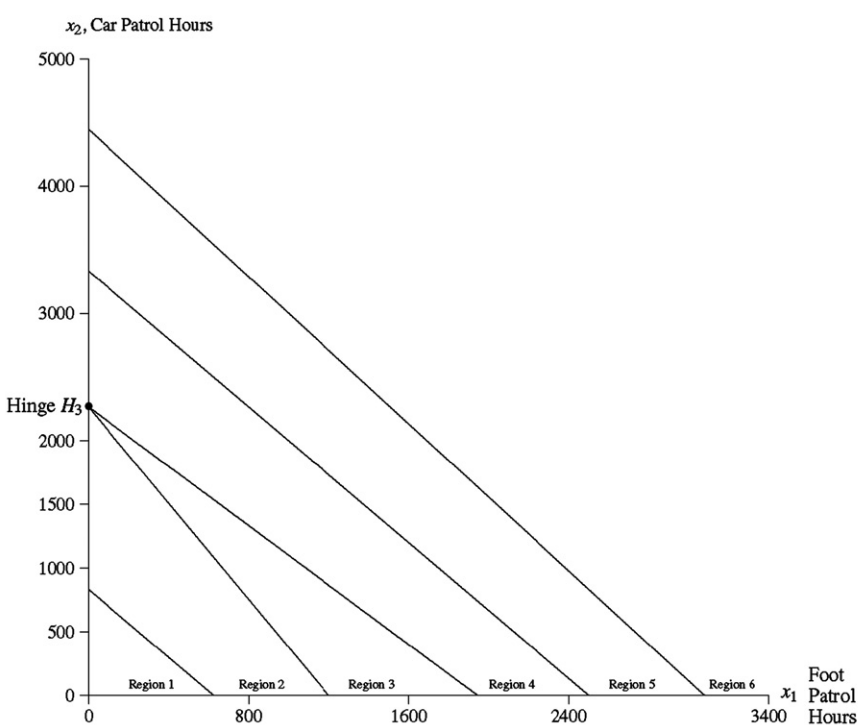

Fig. 1. Attribute space for the police patrol example.