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Online (J, K)-search problem and its competitive analysis

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ABSTRACT

This work proposes an online (J, K)-search problem where an online player has K units of some asset for selling and has to sell at least $J \le K$ units of the asset in a finite number of periods. At the beginning of each period a quoted price is observed and the player has to decide immediately and irrecoverably whether to accept the price as well as the amount of the asset to be sold at the price. The objective is to maximize average selling price. We present two models where at most one unit of the asset can be sold in each period and where one or more units of the asset can be sold in each period. For both models we propose optimal online deterministic algorithms.

1. Introduction

The *online K-search problem* is introduced by Lorenz et al. [10], in which a player has $K \ge 1$ units of some asset to be sold within n periods, aiming at maximizing the total revenue. The value of n is known as a prior. At each period $i = 1, 2, \dots, n$, the player is presented a quoted price p_i and has to *immediately* decide whether to accept the price, i.e., to sell *one unit* of the asset at the price. It is required that all the K units of the asset are sold in the n periods, and thus exactly K out of the n quoted prices are accepted in total. If there are $n - j$ units of the asset left unsold at the end of period j $(n - K < j \le n)$, then the player has to accept all the quoted prices in the remaining periods. Lorenz et al. [10] present an optimal deterministic algorithm as well as an asymptotically optimal randomized algorithm for the problem. Zhang et al. [14] study a variant of the online K-search problem, called the general K-search problem, in which the player may sell one or more units of the asset at each price, and they present an optimal deterministic algorithm for the case with $K < n$.

In this paper, we consider another scenario of the online K-search problem such that the player is required to sell at least J $(1 \le J \le K)$ other than all the K units of the asset within n periods, aiming at maximizing the *average* revenue (or average accepted price). We denote the problem as the online (J, K)-search problem. This is motivated by the observation that in the strategic level of production management, one important goal of a manufacturing enterprise is to ensure its market share during competition. There follows a requirement on the smallest sale amount of the products within a planning time period such as one year or one season. The selling price changes dynamically by the fluctuation of market demand.

One quite related problem is the *online time series search problem* introduced by El-Yaniv et al. [5], in which a player has exactly one unit of the asset to be sold within n periods. Exactly one quoted price is then accepted and it determines the sale revenue. It is proved in El-Yaniv et al. [5] that if quoted prices vary within interval $[m, M]$ $(0 < m < M)$, then an optimal online deterministic algorithm with competitive ratio of $\sqrt{M/m}$ is to accept the first quoted price at least $\sqrt{Mm}$.

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Damaschke et al. [4] study two variants of the online time series search problem where the variation range of quoted prices changes over time. For the first model where the upper and lower bounds of price are of decay speeds, they present an optimal deterministic algorithm and a nearly optimal randomized algorithm. For the second model where the lower bound of price is a constant while the upper bound decreases over time, they propose an optimal randomized algorithm. Xu et al. [12] extend this model by introducing general revenue functions which increase in quoted price for any given period but decrease in time period for any specific quoted price. They propose optimal deterministic algorithms for both cases where the duration n is known or unknown beforehand. Zhang et al. [13] study an online multiple time series search problem in which a player receives one unit of the asset in each period and thus has totally n units for sale in n periods. They focus on the case where the player sells one or more units of the asset and keeps a limited amount of unsold asset at each period, presenting both upper and lower bounds of competitive ratio for the objective of maximizing total revenue.

When the n quoted prices can be ranked and they are presented to the online player in random order such that at any period the player knows the relative ranks of the current and previous quoted prices, it is called the *secretary problem* (see Chow et al. [3]). Babaioff et al. [1] give a framework of generalized secretary problems. The multiple-choice secretary problem (see Kleinberg et al. [9]) and matroid secretary problem (see Babaioff et al. [2]) are similar to the K-search problem such that more than one prices are selected over time. Gharan and Vondrák [7] investigate several variants of matroid secretary problem where the matroid is either known or unknown beforehand. Notice that most of the above work follows a Bayesian framework, and it is assumed that in each period the quoted price follows some given distribution. In this paper we disregard such distribution assumption.

Another related problem is one-way trading where an online player is to exchange dollars to yen in given time periods, in each of which an exchange rate is observed by the player and a decision on converting how many dollars to yen according to the current rate has to be made. The objective is to maximize the amount of yen in the end. El-Yaniv et al. [5] give an optimal online algorithm for the problem, and point out that any online algorithm for one-way trading problem can be viewed as a randomized online algorithm for online time series search problem. Fujiwara et al. [6] study the average performance of online algorithms for one-way trading problem. They propose optimal algorithms considering various optimization measures. Kakade et al. [8] investigate another one-way trading scenario applied in stock market, and named it the price-volume trading problem. A player has an amount of shares of a specific stock at the beginning and the objective is to sell all the shares so as to maximize the average selling price. In this paper, however, it is not required to sell out the asset but there is a lower bound on the amount of sold asset.

In this work we investigate the online (J, K)-search problem such that the player sells at least J units and at most K units of the asset. We focus on two models where at most one unit of the asset is sold at any price and where one or more units of the asset can be sold at any price. For both models, optimal online deterministic algorithms are presented. The rest of this paper is organized as follows. Section 2 describes the online (J, K)-search problem. In Section 3 we focus on the first model where at most one unit of the asset can be sold at any price, and propose an optimal online algorithm. Section 4 presents an optimal online algorithm for the second model where more than one unit of the asset can be sold at a time. Finally we conclude this work in Section 5.

2. Problem statement and preliminaries

2.1. Description of the problem

Online (J, K)-search problem (JKS) A player has K units of the asset to be sold in n periods, and it is required to sell at least J $(1 \le J \le K)$ units of the asset, aiming at maximizing the average selling price. At each period $i = 1, 2, \cdots, n$, the player is presented a quoted price $p_i \in [m, M]$ $(0 < m < M)$ and decides *immediately* the amount of the asset to be sold at the price. A decision of selling none at price p_i implies a rejection of the price. The values of J, K, m, M and n are known to the player at the beginning.

Considering the available amount of the asset to be sold at each quoted price, we propose the following two models JKS_1 and JKS_M.

Model JKS_1: In this model, the player sells at most one unit of the asset for each quoted price, and thus accepts at least J quoted prices in the n periods. If the accumulated amount of sold asset is equal to $J - j$ at the end of period $n - j$ $(0 < j < J)$, then the player has to accept all the quoted prices in the remaining j periods.

Model JKS_M: In this model, one or more units of the asset may be sold at any quoted price, and the player decides in each period whether to accept a quoted price as well as the amount of the asset to be sold at the price. The final amount of sold asset is required to be at least J units.

Notice that if $J = K$, then the model JKS_1 reduces to the online K-search problem in Lorenz et al. [10], and JKS_M reduces to the general K-search problem in Zhang et al. [14]. If $J = K = 1$, then both of the above two models reduce to the online time series search problem in El-Yaniv et al. [5]. Hence, the models to be considered in the paper are generalized variations of that in Lorenz et al. [10] and El-Yaniv et al. [5].