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Performance analysis of model-free PID tuning of MIMO systems based on simultaneous perturbation stochastic approximation

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ABSTRACT

This paper addresses the performance comparison of simultaneous perturbation stochastic approximation (SPSA) based methods for PID tuning of MIMO systems. Four typical SPSA based methods, which are one-measurement SPSA (1SPSA), two-measurement SPSA (2SPSA), Global SPSA (GSPSA) and Adaptive SPSA (ASPSA) are examined. Their performances are evaluated by extensive simulation for several controller design examples, in terms of the stability of the closed-loop system, tracking performance and computation time. In addition, the performance of the SPSA based methods are compared to the other stochastic optimization based approaches. It turns out that the GSPSA based algorithm is the most practical in terms of the stability and the tracking performance.

1. Introduction

PID (proportional-integral-derivative) is most widely used control method in the industry. In order to achieve better control performance, PID design requires a more accurate model of the plant to be controlled. However, it is often difficult to obtain accurate models for the real plants, and it is time consuming to obtain such models even if it is possible. Hence, tuning strategies for PID control parameters based on the I/O data (instead of the plant models) have received considerable attention, which is called model-free PID tuning.

So far, model-free PID control tuning methods have been widely reported for the case with a single-input–single-output (SISO) system. These include metaheuristic approaches such as the particle swarm optimization (Chang, 2009; Gaing, 2004; Jaafar, Mohamed, Abidin, & Ghani, 2012; Zamani, Karimi-Ghartemani, Sadati, & Parniani, 2009), ant colony optimization (Duan, Wang, & Yu, 2006; Hsiao, Chuang, & Chien, 2004), bacterial foraging (Kim, Abraham, & Cho, 2007; Korani, Dorrah, & Emara, 2009), spiral optimization (Nasir, Tokhi, Ghani, & Ahmad, 2012), genetic algorithm (Zhang, Zhuang, Du, & Wang, 2009), simultaneous perturbation stochastic approximation (SPSA) (Yuan, 2008; Xu, Li, & Wang, 2012), simulated annealing (Yachen & Yueming, 2008) and stochastic multi-parameters divergence optimization (Alagoz, Ates, & Yeroglu, 2013). On the other hand, there are a few results for multi-input–multi-output (MIMO) cases. For example, genetic algorithm based method has been presented in Chang (2007) and the controller has been applied to a two-input two-output binary distillation column. Similar works have been reported in Coelho and Mariani (2012), Menhas, Wang, Fei, and Pan (2012b) and Iruthayarajan and Baskar (2009) with the comparative assessment study of various evolutionary algorithms such as binary coded particle swarm optimization algorithms and firefly algorithms. In Menhas, Fei, Wang, and Qian (2012a), cooperative and co-evolving multiple swarms has been proposed for the model-free design of PID controller of a ball mill pulverizing system.

As shown in the above, many approaches use multi-agent based optimization, where the computation times per iteration are proportional to the number of agents. As a result, these methods require heavy computation time in the design process. Hence, it is necessary to develop a tuning strategy which requires less computation time. Meanwhile, it is known that the simultaneous perturbation stochastic approximation (SPSA) is a promising tool from this viewpoint, because it is known to be effective for a variety of optimization problems with less computation time even for high-dimensional parameter tuning.

This paper thus presents a comparative assessment of several SPSA based methods for model-free PID tuning of MIMO systems. In particular, four types of methods, one-measurement SPSA (1SPSA) (Spall, 1997), two-measurement SPSA (2SPSA) (Spall, 1992), Global SPSA (GSPSA) (Yin, 1999) and Adaptive SPSA (ASPSA) (Spall, 2000) are evaluated in some MIMO controller design problems by extensive numerical examples. In order to clarify the benefit of the SPSA based approaches, we consider a higher dimension

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of PID parameters unlike the existing literatures, which have considered not more than 10 PID parameters. Then, the performance of the methods is assessed in terms of the stability of the closed-loop system, tracking performance and computation time. In addition, a comparative assessment between the SPSA based methods and others stochastic optimization based methods, which are Simulated Annealing (SA) (Kirkpatrick, Gelatt, & Vecchi, 1983) and Random Search (RS) (Solis & Wets, 1981), is also presented. So far, there are few papers which purposely discuss on the performance comparison of various PID controller tuning methods for MIMO systems. Therefore, it would be beneficial to present this comparative study and identify the most effective model-free approach for high-dimensional PID tuning.

The remainder of this paper is organized as follows. Section 2 formulates the problem of model-free PID controller tuning. In Section 3, the simultaneous perturbation stochastic approximation based algorithms are introduced. The methods are implemented to several numerical examples with MIMO plants in Section 4. The statistical analysis and comparative assessment are also performed in this section. Finally, some concluding remarks are given in Section 5.

Notation: The symbols $\mathbb{R}$ and $\mathbb{R}_+$ represent the set of real numbers and the set of positive real numbers, respectively. For $\delta \in \mathbb{R}_+$, $\mathrm{sat}_\delta : \mathbb{R}^n \to \mathbb{R}^n$ denotes the saturation function whose ith element given as follows:

$$\text{The } i\text{th element of } \mathrm{sat}_\delta(\boldsymbol{x}) = \begin{cases} \delta & \text{if } \delta < x_i, \\ x_i & \text{if } -\delta \leqslant x_i \leqslant \delta, \\ -\delta & \text{if } x_i < -\delta, \end{cases}$$

where $x_i \in \mathbb{R}$ is the ith element of $\boldsymbol{x} \in \mathbb{R}^n$. The set of $n \times n$ positive definite matrices is denoted by $\mathbb{S}^{n\times n}$.

2. Problem formulation

Consider the MIMO PID control system depicted in Fig. 1 where $\boldsymbol{r}(t) \in \mathbb{R}^q$, $\boldsymbol{u}(t) \in \mathbb{R}^p$, $\boldsymbol{d}(t) \in \mathbb{R}^l$ and $\boldsymbol{y}(t) \in \mathbb{R}^q$ are the reference, the control input, the deterministic disturbance and the measurement, respectively. The plant is the MIMO system $G(s)$. The controller $K(s)$ is given by

$$K(s) = \begin{bmatrix} h_{11}(s) & \cdots & h_{1q}(s) \\ \vdots & \ddots & \vdots \\ h_{p1}(s) & \cdots & h_{pq}(s) \end{bmatrix} \tag{1}$$

for the PID controller

$$h_{ij}(s) := P_{ij}\left(1 + \frac{1}{I_{ij}s} + \frac{D_{ij}s}{1 + (D_{ij}/N_{ij})s}\right), \tag{2}$$

where $P_{ij} \in \mathbb{R}$ is the proportional gain, $I_{ij} \in \mathbb{R}$ is the integral time, $D_{ij} \in \mathbb{R}$ is the derivative time, and $N_{ij} \in \mathbb{R}$ is the filter coefficient.

Next, let us introduce the performance index for the system in Fig. 1. Let

$$\hat{e}_i := \int_{t_0}^{t_f} |r_i(t) - y_i(t)|^2 dt, \tag{3}$$

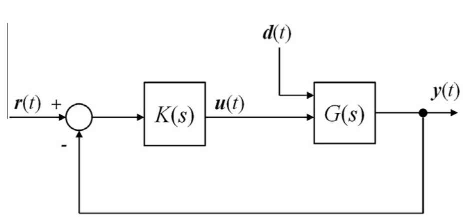

Fig. 1. PID control system.

$$\hat{u}_i := \int_{t_0}^{t_f} |u_i(t)|^2 dt, \tag{4}$$

where $r_i(t)$, $y_i(t)$ and $u_i(t)$ are the ith elements of the vectors $\boldsymbol{r}(t)$, $\boldsymbol{y}(t)$ and $\boldsymbol{u}(t)$, respectively, and $t_0 \in \{0\} \bigcup \mathbb{R}_+$, and $t_f \in \mathbb{R}_+$. The time interval $[t_0, t_f]$ corresponds to the period for the performance evaluation. Then, the performance index is defined as follows:

$$J(\boldsymbol{P}, \boldsymbol{I}, \boldsymbol{D}, \boldsymbol{N}) = \sum_{i=1}^{q} w_{1i}\hat{e}_i + \sum_{i=1}^{p} w_{2i}\hat{u}_i, \tag{5}$$

where $\boldsymbol{P} := [P_{11}P_{12}\ldots P_{pq}]^\top$, $\boldsymbol{I} := [I_{11}I_{12}\ldots I_{pq}]^\top$, $\boldsymbol{D} := [D_{11}D_{12}\ldots D_{pq}]^\top$ and $\boldsymbol{N} := [N_{11}N_{12}\ldots N_{pq}]^\top$, and $w_{1i} \in \mathbb{R}(i = 1, 2, \ldots, q)$ and $w_{2i} \in \mathbb{R}(i = 1, 2, \ldots, p)$ are weighting coefficients, which are given by the designer. Note that p and q are the dimension of the control input $\boldsymbol{u}(t)$ and the measurement $\boldsymbol{y}(t)$, respectively, which are given from the system $G(s)$. The first term in (5) corresponds to the tracking error, while the second means the control input energy. Here, the values of w_{1i} and w_{2i} are selected in a similar way to the standard Linear Quadratic Regulator (LQR) problem. Then, the model-free optimization problem can be described as follows.

Problem 2.1. For the feedback control system in Fig. 1, find a PID controller $K(s)$ which minimizes $J(\boldsymbol{P}, \boldsymbol{I}, \boldsymbol{D}, \boldsymbol{N})$ with respect to $\boldsymbol{P}, \boldsymbol{I}, \boldsymbol{D}$ and $\boldsymbol{N}$ based on the measurement data $(\boldsymbol{u}(t), \boldsymbol{y}(t))$. □

3. PID controller design using simultaneous perturbation stochastic approximation

This section presents a model-free PID tuning method by using SPSA methods.

3.1. Simultaneous perturbation stochastic approximation

Consider the optimization problem given by

$$\min_{\boldsymbol{x}\in\mathbb{R}^n} f(\boldsymbol{x}), \tag{6}$$

where $f : \mathbb{R}^n \to \mathbb{R}$ is the objective function and $\boldsymbol{x} \in \mathbb{R}^n$ is the design parameter. A solution to this problem will be obtained through the simultaneous perturbation stochastic approximation (SPSA). Namely, we try to obtain the solution by the following iterative procedure

$$\boldsymbol{x}(k+1) = \boldsymbol{x}(k) - a(k)g(k) \tag{7}$$

for $k = 0, 1, \ldots$, where $a(k)$ is the gain and $g : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is a function. The function g is summarized as follows.

(i) One-measurement simultaneous perturbation stochastic approximation (1SPSA):

$$g(k) = \begin{bmatrix} \frac{f(\boldsymbol{x}(k)+c(k)\triangle_1(k))}{c(k)\triangle_{11}(k)} \\ \vdots \\ \frac{f(\boldsymbol{x}(k)+c(k)\triangle_1(k))}{c(k)\triangle_{1n}(k)} \end{bmatrix}, \tag{8}$$

where $c(k)$ is the gain and $\triangle_{1i}(k)$ is the ith elements of a random vector $\triangle_1(k) \in \mathbb{R}^n$. Note that some guidance to choose the gain $c(k)$ and the random vector $\triangle_1(k)$ is reported in Spall (1997).

(ii) Two-measurement simultaneous perturbation stochastic approximation (2SPSA):

$$g(k) = \begin{bmatrix} \frac{f(\boldsymbol{x}(k)+c(k)\triangle_1(k))-f(\boldsymbol{x}(k)-c(k)\triangle_1(k))}{2c(k)\triangle_{11}(k)} \\ \vdots \\ \frac{f(\boldsymbol{x}(k)+c(k)\triangle_1(k))-f(\boldsymbol{x}(k)-c(k)\triangle_1(k))}{2c(k)\triangle_{1n}(k)} \end{bmatrix}. \tag{9}$$