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Constraint-activated differential evolution for constrained min–max optimization problems: Theory and methodology

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ABSTRACT

A constraint-activated differential evolution is proposed to solve constrained min–max optimization problems in this paper. To provide theoretical understanding for these problems, their global optima are specified in the proposed definitions. Based on the definition, we propose theorems to prove that a min–max algorithm can be used to solve a max–min problem without any algorithmic changes. Based on the theorems, we propose a constraint-activated differential evolution to solve constrained min–max problems. The proposed method consists of three components, propagation, constraint activation, and inner level evolution. The propagation provides exploitation power of evolution. The constraint activation directly finds a solution which can best activate constraints. The inner level evolution provides continuous evolutionary behavior to prevent convergence premature. The simulation results show that the proposed method attains 100% success rates for all of the numerical benchmarks with an exploitative mutation strategy.

1. Introduction

Differential evolution (DE) (Storn & Price, 1995, 1997; Price, Storn, & Lampinen, 2006) is a simple yet powerful evolutionary algorithm (EA), which is a population-based stochastic search method for global optimization problems in real-parameter search domain. Most research of EA and DE has emphasized mainly on locating a global optimum for a deterministic minimization or a maximization problem (Epitropakis, Tasoulis, Pavlidis, Plagianakos, & Vrahatis, 2011; Guo & Yang, in press). In some real-world optimization scenarios, however, uncertainties are often occurred and unable to avoid. The global optimum might be very sensitive to small variations in parameters (Ong, Nair, & Lum, 2006). Therefore, a preferable solution is probably not the global optimum, but one with high tolerance and robustness to uncertainties. There are many different definitions for a robust solution, such as a solution's expected performance over all possible variations, or a solution's worst performance among all possible disturbances (Paenke, Branke, & Jin, 2006). In this paper, we aim at finding a solution for the best possible worst-case performance, which can be described by a min–max optimization problem.

An unconstrained min–max optimization problem can be solved by coevolutionary algorithms, such as Stackelberg Strategy Coevolutionary Genetic Algorithm (SSCGA) (Basar et al., 1995), Parallel CGA (PCGA) (Herrmann, 1999), Alternating Coevolutionary Particle Swarm Optimization (ACPSO) (Shi & Krohling, 2002), Best Remaining CGA (BRCGA) (Hur, Lee, & Tahk, 2003), Rank-based CGA (RBCGA) (Jensen, 2004), and Aging Sampled GA (ASGA) (Cramer, Sudhoff, & Zivi, 2009). These algorithms have been successfully applied to engineering design problems (Cramer et al., 2009; Guo, Leang Shieh, Chen, & Coleman, 2001a, Guo, Shieh, Lin, & Coleman, 2001b). However, these algorithms are designed without considering non-linear constraints, so that they cannot solve min–max optimization problems with arbitrary constraints. For solving the constrained problem, a constraint-handling method is needed to penalize infeasible solutions. However, there are very limited studies on improvement of the evolutionary algorithm for constrained min–max optimization problems (CMMOPs) in the literature. Some conventional methods had been proposed as follows.

A min–max approach based on modal interval analysis is proposed in Sainz, Herrero, Armengol, and Vehí (2008). The results are obtained by means of simple interval arithmetic computation in simple cases. However, in some complex cases, a branch-and-bound algorithm is required, and the authors have noticed that the complexity increases remarkably when the number of variables grows.

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A Karush–Kuhn–Tucker (KKT) based min–max method is proposed in Lu, Cao, jie Yuan, and Zhou (2008). The method has shown superior performance over the existing methods in the literature. However, the KKT-based method makes assumptions on continuous and convex objective function, as well as available gradient and Hessian information. In real-world optimization problems, the assumptions might not be satisfied, and this limits the applicability of the KKT-based min–max method.

A sequential differential evolution (SDE) approach is proposed in Segundo, Krohling, and Cosme (2012). The algorithm consists of three populations with a scheme of copying individuals to accelerate the convergence. The simulation results show that the success rates in three of the considered four min–max test functions are 100%. However, the known min–max solutions of the test functions are not precise enough (only 10^{-6}), and the success measure is those results lie within 2% of the known optima, which are generally too large to judge the accuracy of the min–max solutions.

In this paper, we propose a constraint-activated differential evolution for constrained min–max optimization problems. In some difficult cases, the number of global optima can be more than one. This motivates our method to locate solutions activating the constraints. In addition, a novel *inner level evolution* and a *propagation* approach are proposed to deal with the multiple optima issue. In the simulation results, the proposed method exhibits superior performance than the SDE over all of the considered three CMMOPs in terms of solution error measure and success rate.

The remainder of this paper is organized as follows. In Section 2, theories of min–max optimization problems are stated. Section 3 briefly introduces differential evolution algorithm, which is employed by the proposed min–max differential evolution as presented in Section 4. The results of experimental analysis are presented in Section 5. Finally, this paper concludes in Section 6.

2. Problem definition

In this section, we define the constrained min–max optimization problem (CMMOP), and propose a series of theorems on the minimization, maximization, min–max, and max–min problems. The proposed theorems provide theoretical understanding of the proposed min–max DE.

Consider the following CMMOP:

$$\min_{\mathbf{x}\in\mathbb{R}^m}\max_{\mathbf{y}\in\mathbb{R}^n} f(\mathbf{x},\mathbf{y}) \tag{1}$$

subject to

$$g_j(\mathbf{x},\mathbf{y}) \leqslant 0, \quad j = 1,2,\ldots,n_c$$

where $f(\mathbf{x},\mathbf{y})$ is an objective function, $\mathbf{x} = (x_1,x_2,\ldots,x_m)^T$ and $\mathbf{y} = (y_1, y_2,\ldots,y_n)^T$ are parameter vectors, $g_j(\mathbf{x},\mathbf{y})$ is the jth inequality constraint, and n_c is the number of inequality constraints. The considered inequality constraints are in less-than expressions, $g_j(\mathbf{x},\mathbf{y}) \leqslant 0$. Equivalently, a greater-than constraint $g_j(\mathbf{x},\mathbf{y}) \geqslant 0$ can be expressed as a less-than constraint $-g_j(\mathbf{x},\mathbf{y}) \leqslant 0$. Also, an equality constraint $h(\mathbf{x},\mathbf{y}) = 0$ can be expressed as two less-than constraints:

$$\begin{cases} h(\mathbf{x},\mathbf{y}) \leqslant 0, \\ -h(\mathbf{x},\mathbf{y}) \leqslant 0. \end{cases} \tag{2}$$

Without loss of generality, we consider the min–max optimization problem only with equal to or less-than constraints in this paper.

The CMMOP in Eq. (1) can also be represented as

$$\min_{\mathbf{x}\in\mathbb{R}^m}\max_{\mathbf{y}\in\mathbb{R}^n} f(\mathbf{x},\mathbf{y}) \tag{3}$$

subject to

$$(\mathbf{x},\mathbf{y}) \in \Omega,$$

where Ω is a subset of $\mathbb{R}^m \times \mathbb{R}^n$ called the *constraint set* or *feasible set*, which is given by

$$\Omega = \{(\mathbf{x},\mathbf{y})|g_j(\mathbf{x},\mathbf{y}) \leqslant 0, \quad \forall j = 1,2,\ldots,n_c\}. \tag{4}$$

Then, the set of all **x**-vectors that may satisfy the constraints is given by

$$\Omega_x = \{\mathbf{x}|(\mathbf{x},\mathbf{y}) \in \Omega, \quad \forall \mathbf{y} \in \mathbb{R}^n\}. \tag{5}$$

Given an **x**-vector, the set of all feasible **y**-vectors is denoted by

$$\Omega_y(\mathbf{x}) = \{\mathbf{y}|(\mathbf{x},\mathbf{y}) \in \Omega\}. \tag{6}$$

With the definitions in Eqs. (5) and (6), the CMMOP in Eq. (3) can be shortly described as

$$\min_{\mathbf{x}\in\Omega_\mathbf{x}}\max_{\mathbf{y}\in\Omega_\mathbf{y}(\mathbf{x})} f(\mathbf{x},\mathbf{y}). \tag{7}$$

To specifically define a global optimum for the CMMOP, we first review definitions of a *global minimizer* and a *global maximizer*. Then, we specify definitions of a *global mini-maximizer* and a *global maxi-minimizer*.

2.1. Minimization and maximization

Definition 1. Suppose that f: $\mathbb{R}^n \to \mathbb{R}$ is a real-valued function defined on a set $\Omega \subset \mathbb{R}^n$. A point $\mathbf{x}^* \in \Omega$ is a *global minimizer* of f over Ω if and only if (iff)

$$f(\mathbf{x}) \geqslant f(\mathbf{x}^*), \quad \text{for all } \mathbf{x} \in \Omega \setminus \{\mathbf{x}^*\}.$$

A *global maximizer* is similarly specified by *Definition* 2.

Definition 2. Suppose that f: $\mathbb{R}^n \to \mathbb{R}$ is a real-valued function defined on a set $\Omega \subset \mathbb{R}^n$. A point $\mathbf{x}^* \in \Omega$ is a *global maximizer*of f over Ω iff

$$f(\mathbf{x}) \leqslant f(\mathbf{x}^*), \quad \text{for all } \mathbf{x} \in \Omega \setminus \{\mathbf{x}^*\}.$$

To shortly represent a *global maximizer* $\mathbf{x}^*$, we write

$$f(\mathbf{x}^*) = \max_{\mathbf{x}\in\Omega} f(\mathbf{x}) \tag{8}$$

and

$$\mathbf{x}^* = \arg\max_{\mathbf{x}\in\Omega} f(\mathbf{x}). \tag{9}$$

Theorem 1. *Let $\Omega \subset \mathbb{R}^n$, f: $\mathbb{R}^n \to \mathbb{R}$ a function on Ω, $\mathbf{x}^*$ a global minimizerof f over Ω, and let $g(\mathbf{x}) = -f(\mathbf{x})$. Then, $\mathbf{x}^*$ is a global maximizerof g over Ω.*

Proof. Because $\mathbf{x}^*$ is a *global minimizer*of f over Ω, we have

$$f(\mathbf{x}) \geqslant f(\mathbf{x}^*), \quad \text{for all } \mathbf{x} \in \Omega. \tag{10}$$

By the above inequality, $f(\mathbf{x}^*)$ is the minimum value of $f(\mathbf{x})$:

$$f(\mathbf{x}^*) = \min_{\mathbf{x}\in\Omega} f(\mathbf{x}). \tag{11}$$

The minimization can be represented as a maximization form. Therefore,

$$f(\mathbf{x}^*) = -\max_{\mathbf{x}\in\Omega}(-f(\mathbf{x})). \tag{12}$$

Since $g(\mathbf{x}) = -f(\mathbf{x})$, we conclude

$$g(\mathbf{x}^*) = \max_{\mathbf{x}\in\Omega}(g(\mathbf{x})) \tag{13}$$

which shows that $\mathbf{x}^*$ is a *global maximizer*of g over Ω. □