



# Chance constrained optimization for targeted Internet advertising<sup>☆</sup>



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## ABSTRACT

We introduce a chance constrained optimization model for the fulfillment of guaranteed display Internet advertising campaigns. The proposed formulation for the allocation of display inventory takes into account the uncertainty of the supply of Internet viewers. We discuss and present theoretical and computational features of the model via Monte Carlo sampling and convex approximations. Theoretical upper and lower bounds are presented along with a numerical substantiation.

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## 1. Introduction

Internet advertising has witnessed growth of 15% in 2012, reaching \$36.6 billion in the United States [11]. This field is markedly different from traditional media used by advertisers such as radio, television and newspaper. Information such as a user's profile, data input and past Internet activity allow marketers to display their advertisements to targeted audiences, resulting in an efficient use of their advertising budget and an improved experience for users.

Our work is interested in the planning of guaranteed display Internet advertising by an ad network, which acts as an intermediary between website publishers and advertisers. Advertisers purchase an advertising campaign from the ad network consisting of a guaranteed campaign goal, which is the number of ads to be displayed, and a set of viewer types, which describes who to show the campaign's ads to. Guaranteed display advertising campaigns are typically for brand awareness where the industry practice is for ad networks to maximize representativeness, which is accomplished by displaying ads of each campaign as proportionally as possible to all targeted viewer types, see Yang et al. [20].

Quadratic optimization programs for this problem have recently been developed by Turner [18] and Yang et al. [20]. In particular, Turner showed that performance metrics are maximized using a proposed allocation methodology assuming the viewer supply follows a certain distribution. Our work addresses the uncertainty in viewer supply using a chance constrained framework. Bharadwaj et al. [4] presented

an extension to [20] tangential to our research, using a two-stage stochastic program with recourse, with the second stage selling or purchasing ads on the spot market if the realized supply is greater or less than expected. An alternative objective to spread ads across campaigns is to maximize entropy, see Tomlin [17]. We pursue the quadratic objective function approach motivated in part by the availability of advanced and efficient solvers.

We introduce the model in Section 2 and formulate the joint chance constrained optimization program to solve the ad network's problem. Section 3 discusses how lower and upper bounds can be found through sample approximations. In Section 4, a convex approximation program is presented which can be used to find lower and upper bounds under different Internet viewer distribution assumptions. We conclude with Section 5, which discusses the results of a computational substantiation of the introduced bounds. A nomenclature table can be found at the end of the text.

## 2. Chance constrained optimization model

### 2.1. Definitions and notation

An online ad network is an aggregator of display ad slots, which it sells to advertisers in partnership with website publishers. For each guaranteed display advertising campaign, the ad network displays ads to a targeted set of viewers that fit certain criteria, such as by demographic or interest. Advertisers are able to choose their targeted set of viewers from the set  $V$  of viewer types, which partitions the publishers' viewers by a predefined set of attributes. Namely, the supply of viewers is modeled as a  $|V|$ -dimensional random variable with mean vector  $\mu$  and covariance matrix  $\Sigma$ . Let  $S_v$  denote the supply of incoming ad slots across all websites in the

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**Nomenclature**

$\alpha$	campaign un-fulfillment tolerance
$\mu$	mean vector of viewer type supply
$\mu_k$	mean vector of the viewer types' supply which campaign $K$ targets
$\mu_v$	mean supply from viewer type $v$
$S_k$	vector of the viewer types' supply which campaign $k$ targets
$S_v$	supply of viewer type $v$
$\Sigma$	covariance matrix of viewer type supply
$\Sigma_k$	covariance matrix of the viewer types' supply which campaign $k$ targets

$\sigma_v$	standard deviation of viewer type $v$ 's supply
$\xi$	campaign un-fulfillment tolerance for (SA)
$g_k$	campaign goal of campaign $k$
$K$	set of advertising campaigns
$K_v$	set of campaigns which target viewer type $v$
$N$	number of viewer type supply scenarios for (SA)
$p_k$	vector of proportions allocated to campaign $k$ from viewer types in $V_k$
$p_{vk}$	proportion of viewer type $v$ 's supply allocated to campaign $k \in K_v$
$V$	set of viewer types
$V_k$	set of viewer types targeted by campaign $k$
$w_k$	campaign $k$ 's priority weighting

ad network loaded by individuals of viewer type  $v \in V$ , with  $\mu_v$  and  $\sigma_v$  being its mean and standard deviation respectively. Let  $K$  denote the set of advertising campaigns. For a campaign  $k \in K$ , the campaign goal  $g_k$  is the number of ads to be displayed to viewers, which we assume is given. For research concerning optimal campaign goal sizes, see [1]. The subset of viewer types  $V_k \subseteq V$  are the viewer types targeted by advertiser  $k$ . The subset of campaigns  $K_v \subseteq K$  are the campaigns which target viewer type  $v$ .

This problem can be viewed as a stochastic transportation problem with each viewer type as a source with random supply and each advertising campaign as a sink with known demand. Each time a user loads a website affiliated with the ad network, a decision must be made as to which advertisement to display. This paper focuses on the high level planning stage at the beginning of each optimization time period, determining what proportion of ads from each viewer type to allocate to each applicable campaign. The decision variables of the ad network are  $p_{vk}$ , the proportion of each viewer type  $v$ 's supply allocated to each campaign  $k \in K_v$ . Another means of planning, especially when dealing with campaigns over short time periods, is by allocating ads to one minute time slots, whereby all visitors during each time period are shown the same ads, see [8].

**2.2. Chance constrained optimization program**

We introduce an optimization program to find the proportion allocations,  $p_{vk}$ , for all viewer types and targeting campaigns, with an explanation following.

$$\begin{aligned}
 \min \quad & \sum_{k \in K} \frac{w_k}{|V_k|} \sum_{v \in V_k} (p_{vk} - q_k)^2 \\
 \text{s.t.} \quad & \sum_{k \in K_v} p_{vk} \leq 1 \quad \forall v \in V \\
 & \mathbb{P} \left( \sum_{v \in V_k} S_v p_{vk} \geq g_k \quad \forall k \in K \right) \geq 1 - \alpha \\
 & q_k = \frac{1}{|V_k|} \sum_{v \in V_k} p_{vk} \quad \forall k \in K \\
 & p_{vk} \geq 0 \quad \forall k \in K, v \in V_k \quad (\text{CC})
 \end{aligned}$$

The first constraint ensures that no more than 100% of a viewer type's supply is allocated. The second constraint models the idea of guaranteed campaign fulfillment, which is interpreted as fulfillment with high probability. In particular, the second constraint ensures that all campaigns are fulfilled with a probability of at least  $1 - \alpha$ , where  $\alpha < 0.5$  is the un-fulfillment tolerance. The fourth constraint ensures that proportions are non-negative.

Chance constrained programming has been used in many different fields such as finance [12] and water resource management [19]. We model campaign fulfillment using a chance constraint for two

reasons. The first is that the success of an advertising campaign is unlikely to change dramatically if  $g_k$  ads or  $(1 - \epsilon)g_k$  ads are displayed for some small percentage  $\epsilon$ , whereas strictly requiring the former may significantly limit the number of advertising campaigns the ad network can accept. With the parameter  $\alpha$ , the ad network is able to balance advertiser satisfaction with the total number of advertising campaigns executed. The second, more fundamental reason is that robust solutions are unlikely to exist without making strong assumptions on the underlying distribution of Internet viewers.  $\mathbb{P}(\cup_{k \in K} \{\sum_{v \in V_k} S_v < g_k\}) = 0$  is a necessary condition for the existence of a robust solution. For distribution assumptions of viewer type supply where this condition does not hold, e.g., normal, Poisson, log-normal, there exists a minimal  $\hat{\alpha} > 0$  such that  $\alpha \geq \hat{\alpha}$  for (CC) to be a feasible program.

The objective of the ad network is to maximize representativeness by allocating each campaign  $k$ 's ads across all  $v \in V_k$  proportionally to the supply, which is achieved by having  $p_{vk} = p_{v'k}$  for all  $v, v' \in V_k$ . Objectives of the following general form have been proposed for guaranteed advertising campaigns, see [18,20],

$$\min \sum_{k \in K} \sum_{v \in V_k} w_{vk} \left( p_{vk} - \frac{g_k}{\mu_k} \right)^2$$

where the  $w_{vk}$ 's are weights,  $\bar{\mu}_k = \sum_{v \in V_k} \mu_v$  is the total expected supply from the viewer types targeted by campaign  $k$ , and  $g_k/\bar{\mu}_k$  is the target proportion. The objective maximizes weighted representativeness of campaigns, assuming the ad network is constrained to fulfill campaigns in expectation. Given the chance constraint, an ideal feasible allocation is unknown a priori. We propose to minimize the variance of each campaign's allocation proportions. The objective is then

$$\sum_{k \in K} \frac{w_k}{|V_k|} \sum_{v \in V_k} (p_{vk} - q_k)^2$$

where  $q_k$  is the mean of the proportions allocated to campaign  $k$  from viewer types in  $V_k$ , enforced in the third constraint, and the weights  $w_k$  represent the campaign's priority to the ad network. For example, assume campaign  $k$  targets 5 viewer types, and a feasible solution to (CC) includes the vector of proportions allocated to campaign  $k$ ,  $p_k = [0.2, 0.3, 0.1, 0.4, 0]$ . Since  $q_k = (0.2 + 0.3 + 0.1 + 0.4 + 0)/5 = 0.2$ , the variance of  $p_k$  is then  $\frac{1}{5} \sum_{v=1}^5 (p_{vk} - 0.2)^2 = 0.02$ . The objective attempts to set  $p_k = [\phi, \phi, \phi, \phi, \phi]$  for some unknown  $\phi$ , which would achieve perfect representativeness for campaign  $k$ , with a variance of 0.

Joint chance constraint programs are in general difficult to solve due to their non-convexity and the numerical integration required to calculate  $\mathbb{P}(\sum_{v \in V_k} S_v p_{vk} \geq g_k, \forall k \in K)$ , see Pagnoncelli et al. [16]. Sample Approximation method provides theoretically well founded solution approaches where Monte Carlo sampling is used to generate approximate mixed integer programs, see [5]. In

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