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Clustering Search and Variable Mesh Algorithms for continuous optimization

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ABSTRACT

The hybridization of population-based meta-heuristics and local search strategies is an effective algorithmic proposal for solving complex continuous optimization problems. Such hybridization becomes much more effective when the local search heuristics are applied in the most promising areas of the solution space. This paper presents a hybrid method based on Clustering Search (CS) to solve continuous optimization problems. The CS divides the search space in clusters, which are composed of solutions generated by a population meta-heuristic, called Variable Mesh Optimization. Each cluster is explored further with local search procedures. Computational results considering a benchmark of multi-modal continuous functions are presented.

1. Introduction

The continuous optimization problems play an essential role in the basic formulation of many real-life problems. Examples include service management (hospital service planning (Lamiri, Grimaud, & Xie, 2009), cab service (Schilde, Doerner, & Hartl, 2011) healthcare planning problems (Nickel, Schröder, & Steeg, 2012 and Shariff, Moin, & Omar, 2012)), industry applications (production planning (Clark, Almada-Lobo, & Almeder, 2011), advance engineering design (Liao, 2010), financial planning (Guillén, Badell, & Puigjaner, 2007), risk management (Papadakos, Tzallas-Regas, Rustem, & Thoms, 2011)), among others. For some decades, the solution approaches for continuous optimization problems have received considerable attention of researchers. In this sense, the main researcher contributions can be classified as either exact or approximate algorithms. Exact methods imply an exhaustive enumeration of the search space, which demonstrate that these methods are time expensive, particularly for non-linear and large-scale optimization problems. For that reason, when problem instance became too large and difficult, approximate methods are often more appropriate even when they do not guarantee an optimal solution, but provide high quality results in reasonable time.

A vast literature can be found about the design and application of approximate algorithms for continuous optimizations problems (see, for instance, Lozano, Molina, & Herrera (2011) for a comprehensive survey). In particular, meta-heuristics have attracted an increasing interest for solving this kind of problems. Meta-heuristics have truly proven to be one the most used alternative for approximately solving complex optimization problems in real context. There exist two main categories of meta-heuristics (Jourdan, Basseur, & Talbi, 2009): single solution algorithms and population-based algorithms. The first category involves local search (Lawler, 1976), Iterated local search (Lourenço, Martin, & Stützle, 2003), Tabu Search (Glover & Laguna, 1997) and Simulated Annealing (Kirkpatrick, Gelatt, & Vechhi, 1983). All these algorithms have as common feature an exploitation mechanism (intensification) when a set of initial solutions is given. However, they may stop at poor quality local minima, which is the main disadvantage of them. The second category gathers the most popular meta-heuristics, Genetic Algorithms (Holland, 1975), Particle Swarm Optimization (Kennedy & Eberhart, 1995), Scatter Search (Glover, 1977), Immune System (Kephart, 1994), Ant Colony Optimization (Dorigo & Stützle, 2010), Variable Mesh Optimization (Puris, Bello, Molina, & Herrera, 2012), Black Hole (Hatamlou, 2013),

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among others. This group of meta-heuristics presents multiple mechanisms to introduce diversity into the population providing better exploration of the solution space. Nevertheless, the aforementioned algorithms are not able to solve the problems with optimality and can still present premature convergence.

Over the last years, a wide variety of hybrid meta-heuristics have been developed (see an exhaustive review in Blum, Puchinger, Raidl, & Roli (2011)). In majority of cases, the best results found are obtained by hybrid approaches, particularly on real-life optimization problems (Dong, 2012; Repoussis, Tarantilis, Bräysy, & Ioannou, 2010; Yu, Yang, & Yao, 2011). Recently another hybrid meta-heuristic was proposed and examined in different optimization problems, the so-called Clustering Search (CS) algorithm (Chaves & Lorena, 2010). The CS devises a general framework to hybridize population based meta-heuristics with clustering in order to detect promising areas before applying local search procedures. CS results have been reached with the application to hard optimization problems, overcoming (or reach equal) the best-known solutions in the literature (Chaves & Lorena, 2011; De Oliveira, Mauri, & Nogueira Lorena, 2012; Nagano, Da Silva, & Lorena, 2012, 2014; Rabello, Mauri, Ribeiro, & Lorena, 2014).

This paper presents CS for real parameter optimization as a hybrid method combined with the Variable Mesh Optimization (VMO) of Puris et al. (2012), called here after CS + VMO, using two local search (LS) algorithms (*Simplex* (Nelder & Mead, 1965) and *Solis-Wets* (Solis & Wets, 1981)). Solutions generated by VMO are clustered and the promising areas detected by CS should be explored through local search as soon as they are discovered. The VMO algorithm considers a population represented by a set of nodes (potential solutions) that are initially distributed as a mesh, and the search process applies expansion and contraction operations in each cycle of the algorithm. The performance of our hybrid method is compared to a VMO alone and VMO + LS implementations according Puris et al. (2012) (for multimodal functions on continuous domains), showing that the combination CS + VMO is superior to the VMO, regarding the solution quality.

The paper is structured as follows. A review of CS and VMO general concepts are described in Section 2. Section 3 describes in detail the proposed hybrid algorithm. Section 4 presents the results through computational experiments, where the CS + VMO performance is evaluated and compared to results of VMO and VMO + LS on a set of test functions. Finally, conclusions are drawn in Section 5.

2. The CS and VMO algorithms

The Clustering Search (CS) (Chaves & Lorena, 2010; Oliveira et al., 2007) is an iterative algorithm that aims to combine meta-heuristics and local search heuristics. The search process gains intensity only in areas of the search space that deserves special attention, called promising regions. Clusters defined by a center c and a radius r frame the search space. The center is a solution that represents the cluster, identifying its location within the search space. Initially, the centers are randomly obtained, but progressively, they tend to fall along truly promising points in the close subspace. The radius r denotes the maximum distance, starting from the center, for which a meta-heuristic solution can be associated to the cluster. The search strategy stimulates intensification, in which solutions of a cluster interact among themselves along the clustering process, generating new solutions (see Fig. 1).

The meta-heuristic is executed independently of the remaining components and must be able to provide a continuous generation of solutions s_k to the clustering process. All clusters are simultaneously preserved to represent these solutions. Similar solutions are collect into groups (assimilated) and the number of clusters is limited by a user-defined upper bound value NC. A distance metric must be defined, a priori, allowing a similarity measure for the clustering process, and a clustering becomes promising when the number of solutions allocated reaches a certain threshold. A local search is applied on the identified cluster center and provides the exploitation of this supposed promising search area.

The VMO is a meta-heuristic with evolutionary characteristics where a set of nodes that represent potential solutions to an optimization problem form a mesh (population) that dynamically grows and moves across the search space (evolves). An expansion process is performed in each cycle, where new nodes are generated in the direction of local extremes (mesh nodes with better quality in different neighborhoods) and the best solution (better quality node obtained throughout the process); as well as starting from the frontier mesh nodes. Then a contraction of the mesh is performed, where in each iteration the best resulting nodes are selected as initial mesh for the next iteration.

The VMO includes the following parameters: population size; number of new nodes required in the expansion process; number of nodes that define the neighborhoods of each node of the mesh and a stop condition (maximum number of fitness evaluations). A detailed algorithm is presented in Puris et al. (2012).

3. The proposed hybrid method CS + VMO

The unconstrained continuous optimization is classically formalized as:

$$min/max\, f(x), \quad x = (x_1, x_2, x_3, \ldots, x_n)^T \in R^n, \tag{1}$$

where x_i is the decision variable defined on the domain $L_i < x_i < U_i$. The terms L_i and U_i are lower and the upper bounds respectively, which are *apriori* defined. Specifically, in this paper, we focus on the multimodal functions (Suganthan et al., 2005), which are harder than the typical unimodal ones.

For the CS + VMO combination some parameters must be defined. Given a solution s_k, generated by VMO, and the centers c_i, a radius r_t are shared for the cluster i in each generation t. The radius for all clusters is determined by

$$r_t = \frac{x_{sup} - x_{inf}}{F \cdot \sqrt[n]{|C_t|}} \tag{2}$$

where x_{sup} and x_{inf} are, respectively, the known upper and lower bounds of the domain of variable x. The dimension of this optimization problem is denoted by n, and $|C_t|$ represents the current number of clusters (initially, $|C_t| = NC$). Depending on the r_t value, one solution generated by VMO could belong to several clusters. Each solution s_k is allocated to the cluster with minimum r_t value relative to its center. Due to the particular evolutionary behavior that presents VMO in solution generation, the user-defined parameter F depends of the number of evaluations reached (CE). More specifically, we tune this parameter according to the maximum number of fitness evaluations C:

$$F = \begin{cases} 6 & if\ \ 0 < CE \leqslant C/4 \\ 10 & if\ \ C/4 < CE \leqslant C/2 \\ 20 & if\ \ C/2 < CE \leqslant 2C/3 \\ 50 & if\ \ 2C/3 < CE \leqslant 3C/4 \\ 100 & if\ \ 3C/4 < CE \leqslant C \end{cases} \tag{3}$$

For each generation of VMO, the clusters densities v_i controls where some clusters become inactive and being eliminated. The local search (LS) is applied in those clusters considered promising according to their densities. The LS is activated, at once, if

$$v_i \geqslant PD \cdot \frac{NS}{|C_t|} \tag{4}$$