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Portfolio optimization in hedge funds by OGARCH and Markov Switching Model

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ABSTRACT

This paper investigates and compares the performances of the optimal portfolio selected by using the Orthogonal GARCH (OGARCH) Model, Markov Switching Model and the Exponentially Weighted Moving Average (EWMA) Model in a fund of hedge funds. These models are used to calibrate the returns of four HFRX indices from which the optimal portfolio is constructed using the Mean-Variance method. The performance of each optimal portfolio is compared in an out-of-sample period and it is observed that overall, OGARCH gives the best-performed optimal portfolio with the highest Sharpe ratio and the lowest risk. Moreover, a sensitivity analysis for the parameters of OGARCH is performed and it shows that the asset weights in the optimal portfolios selected by OGARCH are very sensitive to slight changes in the input parameters.

1. Introduction

The hedge fund industry has grown rapidly in recent years and has become more and more important in alternative investment. According to the twelfth annual Alternative Investment Survey in 2014 by Deutsche Bank [6], hedge funds are estimated to manage assets of more than 3 trillion dollars by the end of 2014. The diversification benefits [28] of hedge funds which allow investors to hold portfolios of other investment funds rather than investing directly in stocks, bonds and other securities are the main factor driving its success [21,18]. However, the 2008 financial crisis caused assets under management to fall sharply because of trading losses and the withdrawal of assets from funds by investors, resulting in a decline of assets by nearly 30% in 2008 [22].

The recent financial turbulence exposed and raised serious concerns about the optimal portfolio selection problem in hedge funds [27]. Many papers have examined portfolio optimization in a hedge fund context [10,30]. The structures of hedge fund return and covariance are crucial in portfolio optimization [5]. The non-normal characteristics of hedge fund returns have been widely described in the literature. Kat and Brooks [19] find that the hedge fund returns exhibit significant degrees of negative skewness and excess kurtosis. According to Getmansky et al. [14] and Agarwal and Naik [1], the returns of hedge fund return are not normal and serially correlated.

Meanwhile, a number of empirical studies show that the correlations of hedge fund return time series are time-varying. Billio et al. [7] and Blazsek and Downarowicz [8] have proposed more Regime-switching models to measure dynamic risk exposures of hedge funds. A non-linear Markov switching GARCH (MS-GARCH) model is proposed by Blazsek and Downarowicz [9] to forecast idiosyncratic hedge fund return volatility. In the other direction, multivariate Garch models are employed to estimate the time-varying covariances/correlations of hedge fund returns. Giamouridis and Vrontos [15] show that the optimal hedge fund portfolio constructed by dynamic covariance models has lower risk and higher out-of-sample risk-adjusted realized return. Harris and Mazibas [17] provide further evidence that the use of multivariate GARCH models optimal portfolios selected by using multivariate GARCH models performance better than static models and also show that exponentially weighted moving average (EWMA) model improves the portfolio performance. Saunders et al. [26] apply Markov-switching model in hedge fund portfolio optimization and show that Markov-switching model outperforms Black–Scholes Model and Gaussian Mixture Model.

In this paper, we extend the results of Saunders et al. [26] to compare the optimal portfolio performances selected by the OGARCH model, the two-state Regime-Switching models and the EWMA Model, using daily observations of HFRX indices for the period 2003–2014. We first detect the exact 2008 financial crisis period by using the Regime-Switching model. Then the out-of-sample portfolio performance in 2007–2009 financial crisis period and the whole sample period are analyzed based on the Sharpe ratio [13] and the mean realized return. Another contribution of the paper is that we

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calculate the asset weight sensitivities in optimal Mean-Variance portfolio to the estimated parameters in OGARCH model.

The rest of this paper is organized as follows. Section 2 introduces the OGARCH model, the Regime-Switching models and the EWMA model. The data set is described and the out-of-sample performance of optimal portfolios is analyzed in Section 3. Section 4 discusses the asset-weight sensitivities with respect to the parameters in the OGARCH Model. Section 5 concludes the paper.

2. Methodology

In this section, we first introduce the different models that can be used to estimate the mean and variance of hedge fund index returns. Then we describe the Mean-Variance portfolio optimization model used for optimal portfolio selection.

2.1. OGARCH model

GARCH models have been applied to volatility modeling with a good success to capture some stylized facts of financial time series, such as fat tails and volatility clustering. One good extension from a univariate GARCH model to a multivariate case is the orthogonal GARCH (OGARCH) model introduced by Ding [12], Alexander and Chibumba [3] and Alexander [2] which is based on univariate GARCH model and principal component analysis. Thereafter, OGARCH has become very popular to model the conditional covariance of financial time series [24]. The OGARCH model is computationally simpler than the multivariate GARCH models for a large dimensional covariance matrix and has achieved outstanding accuracy in forecasting correlation.

In the OGARCH model, the observed time series are linearly transformed to a set of uncorrelated time series by using principal component analysis. The principal component approach is first used in a GARCH type context by Ding [12]. The OGARCH model by Alexander [2] is described as follows.

Let Y_t be a multivariate time series of daily returns with mean zero on k assets with length T with columns $y_1, \ldots, y_k$. We define that the $T \times K$ matrix X_t whose columns $x_1, \ldots, x_k$ are given by the equation

$$x_t = \frac{y_t}{\sqrt{v_i}}$$

where $V = diag(v_1, \ldots, v_m)$ with v_i being the sample variance of the ith column of Y_t.

Let L denote the matrix of eigenvectors of the population correlation of x_t and by $l_m = (l_{1,m}, \ldots, l_{k,m})$ its mth column. l_m is the $k \times 1$ eigenvector corresponding to the eigenvalue λ_m. The column labeling of L has been chosen so that $\lambda_1 > \lambda_2 > \cdots > \lambda_k$. Let D be the diagonal matrix of eigenvalues and $W_m = l_m \sqrt{D}$. The mth principal component of the system is defined by

$$p_m = x_1 l_{1,m} + x_2 l_{2,m} + \cdots + x_k l_{k,m}$$

If each vector of principal components p_m is placed as the columns of a $T \times k$ matrix P, then

$$P = XL$$

The principal component columns are modeled by GARCH(1,1):

$$p_t | \Psi_{t-1} \sim N(0, \Sigma_t)$$

$$p_{i,t} = \epsilon_{i,t}$$

$$\sigma_{i,t}^2 = \gamma_i + \alpha_i \epsilon_{i,t-1}^2 + \beta_i \sigma_{i,t-1}^2 \qquad (1)$$

where Σ_t is a diagonal matrix of the conditional variances of the principal components P. Ψ_{t-1} contains all the information available up to time $t-1$. The conditional covariance matrix of X_n is $D_t = W_n \Sigma_t W_n'$ and the conditional covariance matrix of Y is given by

$$H_t = Var[R_t | \Psi_{t-1}] = \sqrt{V} D_t \sqrt{V} = \sqrt{V} W_n \Sigma_t W_n' \sqrt{V} \qquad (2)$$

The estimation procedure in detail is illustrated in Appendix A for the interested reader.

2.2. Markov switching model

Regime switching models have become very popular in financial modeling since the seminal contribution of Hamilton [16]. Hamilton first proposed the Markov switching model (MSM) to model the real GNP in the US. Since then, these models have been widely used to model and forecast business cycles, foreign exchange rates and the volatility of financial time series.

Suppose the return of I hedge fund indices $i \in \{1, \ldots I\}$ and one global stock index $i=0$ follow a discrete-time Markov switching process. There exists an unobservable process $\{S_t\}_{t=1}^T$ with state space $\{0,1\}$. Each index has a drift and volatility parameter. The log-returns of each index are given by $R_t^i = \mu_{S_t}^i + \sigma_{S_t}^i \varepsilon_t^i$ where $(\varepsilon_t^0, \ldots, \varepsilon_t^I)$ is a multivariate normal with zero mean, unit standard deviation and correlation matrix C_{S_t}. Then the state process S is modeled as a time-homogeneous Markov chain with transition matrix

$$M = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}$$

where $p = \mathbb{P}(S_t = 0 | S_{t-1} = 0)$ and $q = \mathbb{P}(S_t = 1 | S_{t-1} = 1)$ with initial distribution of S as $(r, 1-r)$ with $r = \mathbb{P}(S_1 = 0)$. The parameters in MSM is denoted by the vector $\theta^{MSM} = (p, q, \mu_0, \mu_1, \Sigma_0, \Sigma_1, r)$, for any $p, q \in [0,1]$, $\mu_0, \mu_1 \in \mathbb{R}^{I+1}$, $\Sigma_0, \Sigma_1 \in \mathbb{R}^{(I+1)\times(I+1)}$, r $\in [0,1]$.

The Gaussian mixture model (GMM) is considered as a special case of the Markov-switching model where state process S_t is an iid. random sequence. In this case, the transition probabilities are given by $p = \mathbb{P}(S_t) = 1-r$ and $q = \mathbb{P}(S_t = 0) = r$ with $p+q=1$. Therefore, the state process S is reduced from a Markov chain to a Bernoulli process. The transition matrix becomes

$$M = \begin{pmatrix} p & 1-p \\ p & 1-p \end{pmatrix}$$

The log-returns of each time series are iid. Gaussian mixture random vectors have two components with component weights $(p, 1-p)$. The parameters are denoted by $\theta^{GMM} = (p, \mu_0, \mu_1, \Sigma_0, \Sigma_1)$. For the interested reader, the model estimation procedure is described in Appendix B.

2.3. EWMA model

The exponentially weighted moving average (EWMA) model is very popular among market practitioners. The EWMA model assigns the highest weight to the latest observations and the least to the oldest observations in the volatility estimate.

The variance Σ_t in multivariate EWMA model is defined as

$$\Sigma_t = (1-\lambda)\Sigma_{t-1} + \lambda y_{t-1}' y_{t-1}$$

For each individual element, it is given by

$$\sigma_i^2, j, t = (1-\lambda)\sigma_{i,t-1}\sigma_{j,t-1} + \lambda y_{i,t-1} y_{j,t-1}$$

where λ is the decay factor which determines the importance of historical observations used for estimating the covariance matrix. The value of the decay factor depends on the sample size and varies by asset class. Morgan [23] suggests that a decay factor of 0.94 is used for the daily data set.

Given the decay factor and initial value $\hat{\Sigma}_0$, it is very easy to forecast the conditional covariance matrix. $\hat{\Sigma}_0$ is usually the full