Insurance: Mathematics and Economics 59 (2014) 100–108

Contents lists available at ScienceDirect

Insurance: Mathematics and Economics

journal homepage: www.elsevier.com/locate/ime

Risk aggregation and stochastic claims reserving in disability insurance

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ARTICLE INFO

Article history:
Received January 2014
Received in revised form
August 2014
Accepted 1 September 2014
Available online 10 September 2014

Keywords:
Disability insurance
Stochastic intensities
Conditional independence
Risk aggregation
Stochastic claims reserving
Mimicking

ABSTRACT

We consider a large, homogeneous portfolio of life or disability annuity policies. The policies are assumed to be independent conditional on an external stochastic process representing the economic–demographic environment. Using a conditional law of large numbers, we establish the connection between claims reserving and risk aggregation for large portfolios. Further, we derive a partial differential equation for moments of present values. Moreover, we show how statistical multi-factor intensity models can be approximated by one-factor models, which allows for solving the PDEs very efficiently. Finally, we give a numerical example where moments of present values of disability annuities are computed using finite-difference methods and Monte Carlo simulations.

1. Introduction

The upcoming Solvency II regulatory framework brings many new challenges to the insurance industry. In particular, the new regulations suggest a new mindset regarding the valuation and risk management of insurance products. Historically, premiums and reserves are calculated under the assumption that the underlying transition intensities of death, disability onset, recovery and so on are deterministic. While the estimations should be prudent, this still implies that the systematic risk, i.e. the risk arising from uncertainty of the future development of the hazard rates, is not taken into account. This may have an impact on pricing as well as capital charges. In the Solvency II standard model, capital charges are computed using a scenario-based approach, and the capital charge is given as the difference between the present value under best estimate assumptions, and the present value in a certain shock scenario. As an alternative, insurers may adopt an internal model, which should be based on a Value-at-Risk approach.

Facing these challenges, a plethora of stochastic intensity models have appeared, in particular for modelling mortality. However, these works have largely focused on either calibration or on pricing a single policy under a suitable market implied measure. The risk management aspect has been left largely untouched, although there are notable exceptions. Dahl (2004) derives a pricing PDE for a wide class of life derivatives under a one-factor stochastic intensity model. Dahl points out that shocks from a one-factor model affect all cohorts equally, and that a multi-factor model across cohorts might be more realistic, although it would not offer any further insights. Dahl and Møller (2006) consider pricing and hedging of life insurance liabilities with systematic mortality risk. Biffis (2005) considers annuities pricing under affine mortality models. Ludkovski and Young (2008) consider indifference pricing under stochastic hazard. Norberg (1995) derives an ODE for moments of present values assuming deterministic hazard rates.

While stochastic mortality models have been thoroughly studied in the literature, stochastic disability models have not received the same attention. Levantesi and Menzietti (2012) consider stochastic disability and mortality in the Solvency II context. The approach covers both systematic and idiosyncratic risk, and is suitable for small portfolios. Christiansen et al. (2012) suggest an internal model for Solvency II based on the forecasting technique of Hyndman and Ullah (2007). The approach includes fitting an intensity model over a range of time periods, and fitting a time series model to the time series of parameter estimates. The future development of the intensities is obtained by forecasting or simulation of the time series model.

In this paper, we consider a large, homogeneous portfolio of life or disability annuity policies. The policies are assumed to be independent conditional on an external stochastic process representing the economic-demographic environment. Using a conditional law of large numbers, we show that the aggregated annuity cash flows

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http://dx.doi.org/10.1016/j.insmatheco.2014.09.001
0167-6687/

can be approximated by its conditional expectation, an expression much akin to the actuarial reserve formula. This result highlights the connection between risk aggregation and claims reserving for large portfolios. Further, we derive a partial differential equation for moments of these present values. Moreover, we consider statistical multi-factor intensity models and suggest methods for reducing their dimensionality. Using the so-called mimicking technique introduced by Krylov (1984), we suggest approximating multi-factor models by one-factor models, which allows for solving the PDEs very efficiently. Finally, we give a numerical example where moments of present values of disability annuities are computed using finite-difference methods and Monte Carlo simulations.

The paper is organized as follows. In Section 2, consider an annuity policy under a simple stochastic intensity model. We derive a PDE for computing moments of the random present value of such policies. In Section 3, we examine the aggregated cash flows from a large, homogeneous portfolio of insurance policies, and highlight the connection between risk aggregation and claims reserving. In Section 4, we consider the specific application of disability insurance and show how a class of statistical models can be incorporated into the pricing PDEs. In Section 5, we present numerical results based on disability data from the Swedish insurance company Folksam.

2. Stochastic claims reserving

Let $\tau^1, \tau^2, \ldots$ be random event times (e.g. times of death or recovery from disability), and let

$$N_t^k = I\{\tau^k \le t\}, \quad k \ge 1. \tag{1}$$

Further, define the processes

$$N^k = (N_t^k)_{t\ge 0}, \quad k \ge 1, \tag{2}$$

and let

$$\mathcal{F}^N = (\mathcal{F}_t^N)_{t\ge 0} = (\mathcal{F}_t^{N^1} \vee \mathcal{F}_t^{N^2} \vee \cdots)_{t\ge 0} \tag{3}$$

denote the filtration generated by $N^1, N^2, \ldots$. Now, let Z be a stochastic process with natural filtration $\mathcal{F}^Z = (\mathcal{F}_t^Z)_{t\ge 0}$. Here, N_t^k denotes the state of an insured individual at time t, τ^k represents the corresponding death or recovery time, and Z_t represents the state of the economic-demographic environment. We assume that $N^1, N^2, \ldots$ are independent conditional on $\mathcal{F}_\infty^Z$, and that the $\mathcal{F}^Z \vee \mathcal{F}^N$-intensity of N^k is the process λ^k of the form

$$\lambda_t^k = q(t, Z_t)(1 - N_t^k), \quad t \ge 0. \tag{4}$$

Consider an annuity policy paying $g(t, Z_t)$ continuously as long as $N_t^k = 0$, until a fixed future time T. This type of annuity allows for payments from the contract to depend on time as well as the state of the economic–demographic environment. For example, the contract could be inflation-linked and contain a deferred period. The random present value L_t^k of this policy can be written as

$$L_t^k = \int_t^T g(s, Z_s)(1 - N_s^k) e^{-\int_t^s r(u)du} ds, \tag{5}$$

where the short rate r is assumed to be adapted to $\mathcal{F}^Z$. Further, the time t reserve for this contract is given by $E[L_t^k | \mathcal{F}_t^Z \vee \mathcal{F}_t^N]$, that is, the expected value given the history of the policy and of the environment. Our goal is to find an efficient way to compute this reserve. First, we need the following result, which is given in a slightly different form in Norberg's concise introduction to stochastic intensity models (Norberg, 2010).

Proposition 1. *Assume that $E[|\lambda_t^k|] < \infty$ for each k, $t \ge 0$. Then, for $s \ge t$,*

$$\begin{aligned} E[1 - N_s^k | \mathcal{F}_s^Z \vee \mathcal{F}_t^N] &= P(\tau^k > s | \mathcal{F}_s^Z \vee \mathcal{F}_t^N) \\ &= (1 - N_t^k) e^{-\int_t^s q(u, Z_u)du}. \end{aligned} \tag{6}$$

Proof. First, note that the process $(M_s^k)_{s\ge 0}$ defined by

$$M_s^k = N_s^k - \int_0^s \lambda_u^k du \tag{7}$$

is a $\mathcal{F}^Z \vee \mathcal{F}^N$-martingale (Norberg, 2010, p. 106). For $s \ge t$, let $Y_s^k = P(\tau^k > s | \mathcal{F}_s^Z \vee \mathcal{F}_t^N) = E[1 - N_s^k | \mathcal{F}_s^Z \vee \mathcal{F}_t^N]$. Using (7), we have

$$\begin{aligned} Y_s^k &= E\left[1 - N_s^k + \int_0^s \lambda_u^k du - \int_0^s \lambda_u^k du \,\middle|\, \mathcal{F}_s^Z \vee \mathcal{F}_t^N\right] \\ &= 1 - N_t^k + \int_0^t \lambda_u^k du - E\left[\int_0^s \lambda_u^k du \,\middle|\, \mathcal{F}_s^Z \vee \mathcal{F}_t^N\right] \\ &= 1 - N_t^k + \int_0^t \lambda_u^k du - \int_0^t \lambda_u^k du \\ &\quad - E\left[\int_t^s \lambda_u^k du \,\middle|\, \mathcal{F}_s^Z \vee \mathcal{F}_t^N\right] \\ &= 1 - N_t^k - \int_t^s q(u, Z_u) E[1 - N_u^k | \mathcal{F}_s^Z \vee \mathcal{F}_t^N] du \\ &= 1 - N_t^k - \int_t^s q(u, Z_u) E[1 - N_u^k | \mathcal{F}_u^Z \vee \mathcal{F}_t^N] du \\ &= 1 - N_t^k - \int_t^s q(u, Z_u) Y_u^k du. \end{aligned} \tag{8}$$

Differentiating the above expression, we obtain

$$\begin{cases} dY_s^k = -q(s, Z_s) Y_s^k ds, & s > t, \\ Y_t^k = 1 - N_t^k, \end{cases} \tag{9}$$

with solution $Y_s^k = (1 - N_t^k) e^{-\int_t^s q(u, Z_u)du}$. □

Using Proposition 1, we immediately obtain

$$\begin{aligned} E[L_t^k | \mathcal{F}_t^Z \vee \mathcal{F}_t^N] &= E[E[L_t^k | \mathcal{F}_T^Z \vee \mathcal{F}_t^N] | \mathcal{F}_t^Z \vee \mathcal{F}_t^N] \\ &= E\left[E\left[\int_t^T g(s, Z_s)(1 - N_s^k) \right.\right. \\ &\quad \left.\left. \times e^{-\int_t^s r(u)du} ds | \mathcal{F}_T^Z \vee \mathcal{F}_t^N\right] \middle| \mathcal{F}_t^Z \vee \mathcal{F}_t^N\right] \\ &= (1 - N_t^k) E\left[\int_t^T g(s, Z_s) e^{-\int_t^s q(u, Z_u)du} \right. \\ &\quad \left. \times e^{-\int_t^s r(u)du} ds \,\middle|\, \mathcal{F}_t^Z \vee \mathcal{F}_t^N\right]. \end{aligned} \tag{10}$$

Note that if the environment process Z is replaced by a deterministic function, the functional V_t defined by

$$V_t = \int_t^T g(s, Z_s) e^{-\int_t^s q(u, Z_u)du} e^{-\int_t^s r(u)du} ds \tag{11}$$

corresponds to the time t-reserve of a policy paying g monetary units continuously. Now, since q and g are functions of the stochastic process Z, V_t is a random variable, and the reserve depends on the distribution of V_t. In the case where Z is a Markov process, a natural candidate for the time t reserve of an active