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A new slacks-based measure of Malmquist–Luenberger index in the presence of undesirable outputs ☆

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ABSTRACT

In the majority of production processes, noticeable amounts of bad byproducts or bad outputs are produced. The negative effects of the bad outputs on efficiency cannot be handled by the standard Malmquist index to measure productivity change over time. Toward this end, the Malmquist–Luenberger index (MLI) has been introduced, when undesirable outputs are present. In this paper, we introduce a Data Envelopment Analysis (DEA) model as well as an algorithm, which can successfully eliminate a common infeasibility problem encountered in MLI mixed period problems. This model incorporates the best endogenous direction amongst all other possible directions to increase desirable output and decrease the undesirable outputs at the same time. A simple example used to illustrate the new algorithm and a real application of steam power plants is used to show the applicability of the proposed model.

1. Introduction

One of the most popular methodologies for measuring efficiency of Decision Making Units (DMUs) is the non-parametric frontier mathematical programming approach called Data Envelopment Analysis (DEA). The concept behind DEA is measuring efficiency using production function as initiated in Farrell [25], and later extended to cases with multiple-inputs multiple-outputs by Charnes et al. [10], after which many empirical studies followed [14,15,44].

In measuring efficiency normally inputs have to be minimized and outputs, in vice versa, are maximized. However, in some cases, some (good) outputs should be maximized or some (bad) outputs should be minimized simultaneously. Literature in DEA refers to bad outputs as undesirable factors [45]. One can find a number of undesirable output examples in the empirical literature such as delayed flight [13], poverty rate [7], patient deaths [54], power interruptions and emissions such as SO_x [8,32,59], SO_2, NO_x and CO_2 [34,48,52], NO_x [37,51] CO_x gases [39,55].

One of the variations in DEA studies is the incorporation of undesirable factors in the efficiency measurement, which is termed as eco-efficiency measurement. The incorporation of undesirable factors can be classified into two categories – direct and indirect approaches [43]. The indirect approaches change or customize undesirable factors to include them in the DEA model. On the other hand, the direct approaches treat undesirable as a regular input or output but modify the measurement model. There are several indirect approaches such as taking the additive inverse of undesirable factors [4], treating the undesirable output as an input [52], using multiplicative inverse [31], etc. In contrast, the direct approaches use some theoretical developments such as hyperbolic efficiency model [6], slacks-based measure (SBM) model [49], range adjusted measure (RAM) model [57] and directional distance function [12,16]. Perhaps the most popular approach is the directional distance function (DDF) that has been used in many applications [17,24,40].[1] In spite of its popularity, DDF is known to encounter a problem of infeasibility when it is implemented in types of longitudinal studies to calculate Malmquist–Luenberger index (MLI) [12,23]. The infeasibility problem can occur in mixed period models when a DMU is located beyond the frontier of a different period. Hence, this paper aims to introduce a method to overcome the infeasibility problem of mixed-period DDF models.

The remainder of the paper is organized as follows. In the next section, related literature to DDF models is reviewed and the infeasibility problem is discussed. Section 3 introduces a model with an algorithm to enable DDF type Malmquist Index for

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[1] A comprehensive overview about the ways of treating undesirable outputs can be found in Sahoo et al. [36].

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handling DMUs, which are beyond the efficiency frontier. Section 4 illustrates the proposed algorithm using a numerical example on a real application of steam power plants over eight years. We discuss the results in Section 5. Conclusions and suggestions for future research are given in Section 6.

2. Background and motivation

2.1. Directional Distance Function

In the DEA literature, one of the popular series of models introduced for measuring efficiency/inefficiency is Directional Distance Function (DDF). Using definition of distance Shephard et al. [46] function incorporating undesirable outputs as below:

$$D_o(x,\ y,\ b) = inf\{\varphi : ((y,\ b)/\varphi) \in P(x)\} \tag{1}$$

where $x \in P^I$, $y \in P^J$ and $b \in P^K$ are inputs, outputs and bad outputs of Decision Making Units (DMUs), and φ denotes the expansion or contraction ratio of good and bad outputs, and D_o expands good outputs and contracts bad outputs simultaneously as much as feasible. $P(x)$, production possibility set, is defined as:

$$P(x) = \{(y,\ b) :\ x\ can\ produce\ (y,\ b)\} \tag{2}$$

However, Chung, et al. [12] defines D_o as:

$$\vec{D}(x,\ y,\ b; g) = sup\{\theta : (y,\ b) + \theta g \in P(x)\} \tag{3}$$

where θ plays the same role as φ in (1). Here, g is a vector of directions and is defined as $g=(y,-b)$, using (3), good outputs can be expanded while bad outputs are contracted. Thus, weak disposability implies:

$$(y, b) \in P(x)\ and\ 0 \le \theta \le 1\ imply\ (\theta y, \theta b) \in P(x) \tag{4}$$

But this contradicted with the concept indicting in (3) since weak disposability as in (4) means, to remain feasible, good outputs should be decreased with the same proportion as bad outputs.[2] Free disposability is also written as below:

$$(y, b) \in P(x)\ and\ y' \le y\ imply\ (y', b) \in P(x) \tag{5}$$

This also implies that good and bad outputs are freely disposable. In addition, it is also assumed that good and bad outputs are produced jointly namely "null-joint", which means, it is not possible to produce good output without producing any bad output.

Now according to Chung et al. [12] $P(x)$ can be rewritten as below to be compatible with (2)–(5):

$$P(x) = \Big\{(y,\ b) : \sum_{n=1}^{N} z_n x_{in} \le x_{io} \quad i=1,2,\dots,I; \quad \sum_{n=1}^{N} z_n y_{jn} \ge y_{jo} + \theta y_{jo}$$
$$j=1,2,\dots,J; \quad \sum_{n=1}^{N} z_n b_{kn} = b_{ko} - \theta b_{ko} \quad k=1,2,\dots,K; \ z_n \ge 0; \ n=1,2,\dots,N\Big\} \tag{6}$$

here z_n are intensity variables. According to (6) the following linear programming model can be used to find $\vec{D}(x,\ y,\ b;g)$, $g=(y,-b)$:

$$\begin{aligned} &\vec{D}_o(x,\ y,\ b;g) = Max\theta \\ &\text{Subject to} \\ &\sum_{n=1}^{N} z_n x_{in} \le x_{io}; \quad i=1,2,\dots,I \\ &\sum_{n=1}^{N} z_n y_{jn} \ge y_{jo} + \theta y_{jo}; \quad j=1,2,\dots,J \\ &\sum_{n=1}^{N} z_n b_{kn} = b_{ko} - \theta b_{ko}; \quad k=1,2,\dots,K \\ &z_n \ge 0; \quad n=1,2,\dots,N \end{aligned} \tag{7}$$

Chambers et al. [9] defined a similar model without considering undesirable outputs as formulated in Model (8) below:

$$\begin{aligned} &\vec{D}_o(x,\ y;g) = Max\theta \\ &\text{Subject to} \\ &\sum_{n=1}^{N} z_n x_{in} \le x_{io} - \theta x_{io}; \quad i=1,2,\dots,I \\ &\sum_{n=1}^{N} z_n y_{jn} \ge y_{jo} + \theta y_{jo}; \quad j=1,2,\dots,J \\ &z_n \ge 0; \quad n=1,2,\dots,N \end{aligned} \tag{8}$$

Here g equals $(y,-x)$. It is worthwhile to note that, third series of constraints in Model (7) (which are corresponding to the bad outputs, b's) are similar to the first series of constraints in Model (8) (which are corresponding to inputs, x's) whereas in Model (7) third series of the constraints are equalities.

As indicated in Fukuyama and Weber [27] and Zhou et al. [58] a conventional DDF model may overestimate the efficiency when non-zero slacks appears in the efficiency measures, hence, a new generation of non-radial DDF model has been introduced to the DEA literature [28] and have been successfully applied in many applications [26,36,42,53,58]. DDF models have also been applied in many disciplines including energy efficiency [24], assessment of banks [3], agriculture [5]. Recently Färe and Grosskopf [19] have investigated affine data translation properties of DDF models. In Section 3 we discuss the non-radial DDF Models in details.

2.2. Malmquist–Luenberger index

Based on the Malmquist index approach for efficiency and technology change, Chung et al. [12] developed the Malmquist-Luenberger index (MLI). The MLI incorporates undesirable outputs, to evaluate productivity change when a longitudinal study is conducted.[3] In the same manner as Malmquist index which is calculated using a series of DEA models [21]; the MLI deploys Directional Distance Function to solve various linear problems for decomposing MLI to technology and productivity change during the period of study.

Now we address how Model (7) can be used to calculate the following components of MLI in the longitudinal studies:

$$ML_t^{t+1} = \left[\frac{\left(1+D_o^t\left(x^t, y^t, b^t; y^t, -b^t\right)\right)}{\left(1+D_o^{t+1}\left(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1}\right)\right)} \times \frac{\left(1+D_o^{t+1}\left(x^t, y^t, b^t; y^t, -b^t\right)\right)}{\left(1+D_o^t\left(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1}\right)\right)}\right]^{1/2} \tag{9}$$

where $t=1,\dots,T$ denotes periods of study. In other words, $D_o^{t+1}(x^t, y^t, b^t; y^t, -b^t)$, for example, represents the distance function for frontier in period $t+1$ while assessing a DMU from period t.

Therefore, the linear programs corresponding to $D_o^{t+1}(x^t, y^t, b^t; y^t, -b^t)$ and $D_o^t(x^{t+1}, y^{t+1}, b^{t+1}; y^{t+1}, -b^{t+1})$ are named mixed period models, since the DMU under assessment and the frontier are from different periods. This can lead to an infeasibility problem, which is discussed further in Section 2.4.

2.3. Slacks-based measure of inefficiency

The slacks-based measure of inefficiency as introduced by Tone [49] is one the most common model applied in DEA. Tone [50] has also deployed the slacks-based measure and its variations to measure productivity factors. Further, Färe and Grosskopf [18]

[2] Economic implications of the weak disposability axiom is further discussed in Kuosmanen and Kazemi Matin [35].

[3] It should be noted that MLI is not the only index for evaluating productivity change in longitudinal studies in the presence of undesirable factors, researchers have introduced alternative Malmquist indexes, such as Malmquist CO_2 emission performance index (MCPI) [56] or Environmental Performance Index (EPI) [33].