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The predictive density simulation of the yield curve with a zero lower bound

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ABSTRACT

Since Diebold and Li (2006) proved the outstanding performance of a three-factor Gaussian dynamic Nelson–Siegel (DNS) model in forecasting the U.S. yield curve, the DNS model and its variants have been widely applied in many areas of macroeconomics and finance. However, despite its popularity one practical problem with the DNS approach is that it produces a substantially high probability of negative future short-term government bond yields for the recent financial crises. In this study, we provide predictive densities for yield curves that have, in general, non-negative support. To this end, we propose and estimate a new DNS model that takes a zero lower bound into account. In the model, the yields are determined as a linear function of the vector-autoregressive factors, which is constrained to be non-negative. We employ a Bayesian econometric approach for estimation and density forecasting. As a result of the zero lower bound restriction, the Gibbs-sampling method is no longer applicable, unlike in standard DNS models. Instead, we propose an efficient Markov chain Monte Carlo method, and demonstrate that the non-negative predictive yield curve density, as well as the model parameters and factors can be simulated with high efficiency. Moreover, we find that, for the U.S. yield curve, the Svensson four-factor DNS model with a zero lower bound is most preferred among the alternatives we consider.

1. Introduction

The Nelson–Siegel model of the term structure of interest rates has been broadened into a dynamic common factor model in which the latent factors are usually interpreted as the time-varying level, slope, and curvature of the yield curve.[1] Since Diebold and Li (2006) proved the outstanding performance of the three-factor dynamic Nelson–Siegel (DNS(3)) model in terms of both forecasting and in-sample fit, the model has been widely used by both practitioners and policy makers.[2] Recently, many studies have extended the DNS(3) model to improve its forecasting performance. For instance, de Pooter (2007) examines the forecasting ability of DNS models using different numbers of factors and finds that predictive gains can be obtained by adding a second slope factor to the DNS(3) model. Zantedeschi et al. (2011) allow for an unknown number of change points in the yield curve dynamics, which helps the model outperform the random-walk model at three-month-ahead and six-month-ahead forecasts.

Despite such significant advances in DNS modeling, we have faced a new challenge in forecasting the U.S. yield curve since the end of 2008, when the federal funds rate was set close to zero. Fig. 1 plots the time series of bond yields with various maturities. The short-term bond yield seems to have been substantially constrained by the zero lower bound (ZLB) from December 2009 onward. The ZLB

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[1] See Diebold et al. (2006).

[2] According to the Bank for International Settlements (BIS, 2005) report, nine of thirteen central banks use the three-factor dynamic Nelson–Siegel model, or its modified versions to fit and construct zero-coupon bond yield curves.

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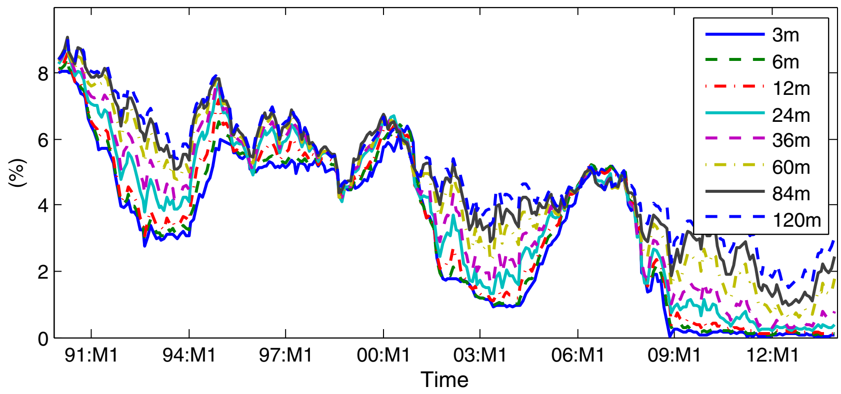

Fig. 1. Time series of the U.S. government bond yields. The time span is January 1990 to December 2013.

also appears to have affected one- and two-year maturity U.S. Treasury bond yields since August 2011, when the Federal Reserve's Open Market Committee announced that the exceptionally low federal funds rate would continue until at least the mid-2013. These unprecedented low bond yields raised a new practical issue. Here, the support of the predictive densities generated by the DNS(3) model, especially for short-term yields, includes the negative region, as shown in Fig. 2. This figure presents the one-month-ahead predictive density of the U.S. yield curve produced by the DNS(3) model for the forecast period of December 2013. The point forecasts of the yield curve are non-negative. However, in terms of the density prediction, the forecasts are problematic because the 95% credibility intervals include negative values for the short- and mid-term bond yields. Specifically, the simulated posterior predictive density indicates a 22.3%, 27.7%, 36.0%, and 14.4% probability of negative one-month-ahead three-month, six-month, one-year, and two-year bond yields, respectively. This problem arises because the model-implied yields are Gaussian, although the ZLB is no longer negligible.

The primary goal of this study is to provide an econometric framework for simulating predictive densities that are applicable even when future bond yields are expected to be near the ZLB. To this end, we propose and estimate a new DNS model with a ZLB constraint on the yield curve dynamics. In the standard DNS model, each of the bond yields is basically determined by a deterministic linear function of time-varying continuous factors. Although the maturity-specific normal errors, which are independent of the factors, are assumed to be present, they are negligible when the number of factors is large. The exogenous factors are typically assumed to follow a first-order Gaussian vector autoregressive process, VAR(1). In this study, we incorporate a ZLB into the standard DNS model. In our model, we assume that the factors are generated from a truncated joint normal distribution within the range where the linear function of the factors is non-negative, conditioned on the lagged factors. This ZLB restriction on the linear function of the factors ensures the non-negativity of the model-implied yields.

Our econometric approach is Bayesian. The model parameters and factors are simulated from their joint posterior distributions with a ZLB constraint. Then, using these model parameters and factors, we simulate the posterior predictive distributions for the future yields. A Gibbs-sampling method is no longer applicable because of the ZLB restriction. We thus propose a Markov chain Monte Carlo (MCMC) method based on a Metropolis–Hastings (M–H) algorithm, and show that the parameters and factors can be simulated with high efficiency. In this paper, the k-factor DNS model with the ZLB is referred to as the NDNS(k) model. We then compare the out-of-sample yield curve density prediction performance of the NDNS $(k)_{k=3,4}$ models to that of the DNS $(k)_{k=3,4}$ models.

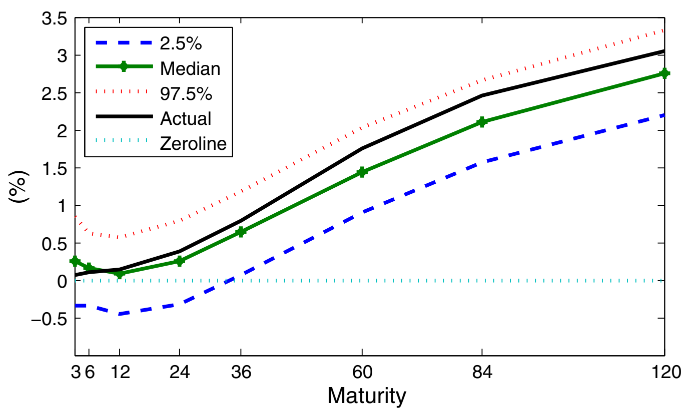

Fig. 2. One-month-ahead predictive density from the DNS(3) model. This figure plots the one-month-ahead predictive distribution of the U.S. yield curve from the three-factor DNS model. The forecast period is December 2013.