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Diffusion of two brands in competition: Cross-brand effect

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HIGHLIGHTS

- Equilibrium points of Bass's model equations system for brand competition.
- Only equilibrium points associated to the market saturation exist.
- Bass's equation system and an agent-based model are compared.
- Crossed coefficients are necessary to perform the correspondence between both models.
- Study of cross-brand effect in the product diffusion.

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ABSTRACT

We study the equilibrium points of a system of equations corresponding to a Bass based model that describes the diffusion of two brands in competition. To increase the understanding of the effects of the cross-brand parameters, we perform a sensitivity analysis. Finally, we show a comparison with an agent-based model inspired in the Potts model. Conclusions include that both models give the same diffusion curves only when the cross coefficients are not null.

1. Introduction

It is of present interest, from an economic point of view, to fully understand the processes and drivers behind the diffusion of innovations. The existence of many recent papers reviewing this subject, such as [1–3] is an example of its relevance. A pioneer work is the well-known Bass model [4] which describes the curves of adoption for many durable goods with great precision.

Keeping in mind the success of the Bass model in the description of the diffusion process of many new products, it is natural to extend the formalism to describe the adoption curves of two brands which are launched simultaneously and dispute the same market. This possibility has been previously investigated, as we can see in Refs. [5,3].

We have, within the Bass formalism, two coupled differential equations. The simplest coupling is the one associated to the competition between two brands within a common market. However, as in Ref. [6], a cross-brand effect can be introduced

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that takes into account the interactions between the adopters of one brand with the potential adopters of the other brand. This adds new coupling parameters in the equation that further describe the dynamics of product adoption, these will be referred to as cross-brand parameters from here on.

The problem of two brands in competition can also be approached from a microscopic point of view, where both the preferences of each individual and its interaction with others are taken into account. In a previous work [7] it was shown, for an innovation diffusing in a market, that it is possible to relate the microscopic variables of an agent-based model (ABM), to the parameters of the Bass model. For that purpose, a physical analogy was used, namely, the well-known statistical Ising model was adapted for the study of technology diffusion [8].

Later on, a generalization for many options was described [9], using an analogy with the statistical Potts model [10] which lets us consider "n" options in competition. There are other approaches for modeling competition between brands, such as the case of Ref. [3] cited before. Also, in Ref. [11] the problem of competition between two brands is studied in terms of: "innovate or copy". There, the algorithm comes from the Logit model which, because of the threshold condition used, is analogous to our Potts model with "zero temperature". Another example of an agent based model for competition between options is the one used in Ref. [12]. In that paper the Monte Carlo method is used and companies (agents), can choose between two options (products or services) by means of a mechanism based on costs and payoffs.

One of the goals of the present work is to relate the microscopic variables of the agent-based model to the macroscopic variables of the Bass based model for the case of two brands in competition. In particular, we focus our research in a systematic study of the cross-brand terms, looking for the values that fit the ABM for two brands. We do this with the hypothesis that, except for the macroscopic parameters associated to the cross-brand effect, all the other parameters can be equal to the ones corresponding to the isolated brands (without competition). Although the best fit between the microscopic and macroscopic models is achieved by varying all of the microscopic variables, this hypothesis can be considered approximately valid, which encourages the generalization of the line traced in Ref. [8] regarding the process of correspondence between the macroscopic and microscopic models.

This paper is organized as follows: In Section 2, we introduce the system of two differential equations that describes the dynamics of two brands in competition, we perform the analysis of the equilibrium points of the system and show the influence of the value of the parameters on those points. In Section 3, we provide the n-optional formalism used in the ABM. In Section 4, we perform the comparison between the dynamics emerging from the two considered models (i.e. Bass-like and ABM). In Section 5, we summarize the main conclusions.

2. Coupled Bass system

In the original Bass article [4] the aggregate adoption rate of a new product (consumer durable good), in a given potential market (m) is calculated as a function of two kinds of parameters, each describing two different types of influence: the innovation parameter (p) reflects people's intrinsic tendency to adopt an innovation, while the imitation parameter (q) reflects the "word of mouth" or the "social contagion", representing the positive influence that people that has already adopted makes on potential adopters.

Such as stressed in Ref. [5], when two brands are considered, we can identify two kinds of effects related to the interaction between adopters and potential adopters: one is known as "within-brand" and the other as "cross-brand". The first one is the influence of the adopters of a brand on the probability that potential adopters will adopt the same brand. The second is the positive effect produced by adopters of a brand on the probability that potential adopters will adopt the other brand. Libai observed the cross-brand effect in Apple's launch of the iPhone in 2007 where word-of-mouth transmission of the product's particularities incentivated the sales not only of the iPhone but of the whole smartphone category.

Using the same equations than Ref. [5] we have

$$\frac{\mathrm{d}N_1}{\mathrm{d}t}(t) = \left[p_1 + q_{11}\frac{N_1(t)}{m} + q_{12}\frac{N_2(t)}{m}\right](m - N), \tag{1}$$

$$\frac{\mathrm{d}N_2}{\mathrm{d}t}(t) = \left[p_2 + q_{22}\frac{N_2(t)}{m} + q_{21}\frac{N_1(t)}{m}\right](m - N) \tag{2}$$

where N_1 and N_2 are respectively the number of adopters of brands 1 and 2, m is the common potential market, $N(t) = N_1(t) + N_2(t)$ is the total number of adopters at time t, p_1 and p_2 are the external influence parameters for brands 1 and 2 respectively, q_{11} and q_{22} are the within-brand influence parameters for brands 1 and 2 respectively, q_{12} is the cross-brand influence of brand 2 on brand 1 and conversely q_{21} of brand 1 on 2.

Such as indicated in Ref. [5] there is some bibliography related with the last formulation where the approach $q_{ii} = q_{ij} = q_{ji}$ or even $q_{ij} = 0$ is used. However, those coefficients are never considered to be negative. A negative value of the cross-brand term would mean that the consumer that adopted brand 2 for example, would generate a positive influence to adopt brand 2 on potential adopters, but this would be redundant, since that effect is already considered in Eq. (2) through q_{22}. On the other hand, a positive value for the cross-brand term would mean a reinforcement for the purchase of brand 1 by adopters of brand 2 and this would make sense if these adopters have had a bad experience with the said brand. Therefore the only restriction that we employ on the cross coefficients is $q_{ij} \geqslant 0$.