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Existence and efficiency of oligopoly equilibrium under toll and capacity competition

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ABSTRACT

This paper studies the existence and efficiency of oligopoly equilibrium under simultaneous toll and capacity competition in a parallel-link network subject to congestion. We establish sufficient conditions for the existence of a pure-strategy oligopoly equilibrium and then derive upper bounds on the efficiency loss of the oligopoly equilibrium over the social optimum under different inverse demand function assumptions, respectively. Furthermore, we show that these bounds are demand-function free and only dependent upon the number of competitive roads.

1. Introduction

With the increasing of private involvement in highway supply, several asymmetric toll roads serving for one origin–destination pair have been often seen all over the world, especially, in many developing countries (Roth, 1996; De Palma and Lindsey, 2002; Yang and Woo, 2000). For private operators of these competitive roads, to benefit financially from their participation involves two fundamental decision variables, i.e., the toll charge and the road capacity (Verhoef and Rouwendal, 2004; De Borger and Van Dender, 2005). The charged toll not only partly determines the total revenue that the private operator can obtain, but also to a certain extent reduces the number of road users by internalizing the congestion externality among them. The larger capacity can reduce the congestion level of a road and then increase the toll revenue due to more road usage, but it inevitably increases the construction cost of the road. As a result, the charged toll and the selected capacity both affect the private operator's profit either positively or negatively.

In this paper, we study a simultaneous toll and capacity game among multiple private road operators in the aforementioned parallel-competitive environment with elastic demand. More specifically, under the beliefs about the others' decisions, each profit-maximizing operator is assumed to simultaneously determine its toll charge and road capacity subject to the resulting traffic flow pattern in equilibrium. Given the charged tolls and the selected capacities, which road a traveler chooses to travel is determined by the generalized travel cost, i.e., the sum of the charged toll and the experienced travel time cost. Briefly speaking, our study's objective is to establish sufficient conditions for the existence of a pure-strategy oligopoly equilibrium, and then to provide upper bounds on the efficiency loss from the interaction between selfish users and profit-maximizing operators when competition substitutes for government regulation.

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This paper is related to the literature in transportation science on quantifying the inefficiency of selfish travel behavior (Roughgarden, 2003; Roughgarden and Tardos, 2002; Correa et al., 2004, 2008; Guo et al., 2010; Liu et al., 2009; Yang et al., 2010). The term "price of anarchy", first introduced by Koutsoupias and Papadimitriou (1999), has been often coined to characterize the degree of inefficiency, which is the worst-possible ratio between the total system cost under the user equilibrium (UE) flow pattern and that under the system optimum (SO) flow pattern. Readers may refer to Roughgarden (2005) and Yang and Huang (2005) for recent reviews. Besides that resulting from selfish travel behavior, the profit-driven efficiency loss under oligopoly competition will be also considered in this paper.

There are a few studies focused on toll competition among congested markets for investigating the inefficiency resulting from both selfish routing and oligopoly competition. Considering the case of inelastic demand, Acemoglu and Ozdaglar (2007) showed that a change in the market structure from monopoly to duopoly in a two-link network typically increase the inefficiency, and a pure-strategy equilibrium always exists when travel cost functions are linear. For their simplified case, they characterized a tight bound of 6/5 on inefficiency in pure-strategy equilibria with zero latency at zero flow and a tight bound of $1/(2\sqrt{2}-2)$ with nonzero latency at zero flow. In contrast, Ozdaglar (2008), Hayrapetyan et al. (2007), and Engel et al. (2004) studied toll competition among profit-maximizing oligopolists with elastic demand. All of them discussed the existence of a pure-strategy equilibrium under the assumption that the demand function is concave. Besides, Hayrapetyan et al. (2007) also provided non-tight upper bounds on the inefficiency, in particular, arrived at 5.064 when linear cost functions and concave demand functions are considered. However, under a similar environment, Ozdaglar (2008) found that the tighter bound on the inefficiency only has 1.5. Engel et al. (2004) pointed out that the increase in competition can improve the efficiency even if demand is kept proportional to the network size. Xiao et al. (2007a) examined the inefficiency of toll competition with general inverse demand functions, but did not explicitly discuss the existence of oligopoly equilibrium. In their work, the parameterized general bounds on the inefficiency turn out to be more complicated and dependent upon the specific property of the inverse demand function.

Recently, Acemoglu et al. (2009) and Xiao et al. (2007b) studied the efficiency of oligopoly equilibrium in a toll and capacity competition game. In particular, Acemoglu et al. (2009) considered a two-stage game with fixed total demand and capacity constraints, where the operators first choose road capacities and then set toll charges. Comparably, Xiao et al. (2007b) indeed analyzed a one-shot game where operators compete by simultaneously choosing tolls and capacities. Xiao et al. (2007b) found that if a pure-strategy oligopoly equilibrium exist, the inefficiency of simultaneous toll and capacity competition can be upper bounded by a parameterized expression in relation to the ratio of the realized demand at oligopoly equilibrium over that at social optimum. However, they did not make a detailed analysis of the existence of the oligopoly equilibrium. In addition, their observations, that the inefficiency bound declines with the number of competitive roads, are mainly made for symmetric roads with the exponential and linear demand types.

Distinguished from previous work, the main contributions in this paper focus on the following two aspects. First, a pure-strategy oligopoly equilibrium may not exist in a general one-shot game. To obtain sufficient conditions for its existence, we do not assume the concavity of inverse demand function, $B(V)$, a restrictive assumption often made in the related literature, e.g., Engel et al. (2004) and Johari et al. (2010). In contrast, we show that if $VB(V)$ is a concave function of the total number of users, V, and all link travel time cost functions are linear, a pure-strategy equilibrium exists. The concave $VB(V)$ assumption captures some widely used demand functions besides the concave types, such as exponential ones (Guo and Yang, 2009; Tan et al., 2010). In other words, it holds for most common cumulative distribution functions representing users' willingness-to-pay for travel (see Remark in Section 5), e.g., normal, uniform, exponential, and Pareto distributions. Many of them are convex, or concave–convex. Second, based on the concave assumption of $VB(V)$ and considering its three special cases, i.e., that inverse demand functions are concave, log-concave or have the increasing elasticity, respectively, we derive the upper bounds on the efficiency loss of oligopoly equilibrium. We also show that these bounds are demand-function free and only dependent upon the number of competitive roads.

This paper is organized as follows. Section 2 introduces some notation and assumptions used throughout this paper. In Sections 3 and 4, we present the oligopoly equilibrium and social optimum models, respectively. Section 5 bounds the efficiency loss of oligopoly equilibrium. Section 6 concludes the paper.

2. Notation and assumptions

We consider $n \geqslant 2$ parallel toll roads connecting a common origin and destination. Let $N = \{1,2,\ldots,n\}$ denote the set of roads. Each toll road is assumed to be independently financed and operated by different firms. These operators compete for users by choosing toll charge, τ_i, and road capacity, s_i, $i \in N$. Let v_i be the number of users served by operator i. The total revenue of operator i can be denoted by $\tau_i v_i$. To obtain some meaningful insights, we assume that there are constant returns to scale in road construction and daily maintenance or other operating costs can be internalized through toll charging. Thus, the profit of operator i is given by

$$\pi_i(\tau_i, s_i) = \tau_i v_i - k_i s_i, \quad i \in N, \tag{1}$$

where constant $k_i > 0$ denotes the unit construction cost for road capacity.

For travelers, the generalized travel cost of making one trip on road i consists of the charged toll, τ_i, and the experienced travel time cost (we consider homogeneous users only and assume a unity VOT for simplicity), $t_i(v_i,s_i) \geqslant 0$, dependent upon