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Multifractal properties of price change and volume change of stock market indices

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ABSTRACT

We study auto-correlations and cross-correlations of daily price changes and daily volume changes of thirteen global stock market indices, using multifractal detrended fluctuation analysis (MF-DFA) and multifractal detrended cross-correlation analysis (MF-DXA). We find rather distinct multifractal behavior of price and volume changes. Our results indicate that the time series of price changes are more complex than those of volume changes, and that large fluctuations dominate multifractal behavior of price changes, while small fluctuations dominate multifractal behavior of volume changes. We also find that there is an absence of correlations in price changes, there are anti-persistent long-term correlations in volume changes, and there are anti-persistent long-term cross-correlations between price and volume changes. Shuffling the series reveals that multifractality of both price changes and volume changes arises from a broad probability density function.

1. Introduction

Price and trading volume are two key variables in finance. Understanding how changes in volume affect changes in price adds valuable insight on the structure (e.g. how the rate of information flow affects price dynamics) and complexity of financial markets [1]. The relationship between trading volume and security prices has been studied for over 40 years, since Osborne hypothesized in 1959 that security prices can be modeled as a diffusion process with its variance dependent on the number of transactions [2]. Ying analyzed six years of daily price and volume for the New York Stock Exchange (NYSE) and found that: (i) a small (large) volume is usually accompanied by a fall (rise) in price, and (ii) a large increase in volume is usually accompanied by a large rise or fall in price. This suggests there are positive correlations between volume and the magnitude of price change which was later confirmed in various studies [3–5].

Several techniques have been developed to study fractal and multifractal properties in time series [6–13], where multifractal detrended fluctuation analysis (MF-DFA) [10] and multifractal detrended cross-correlation analysis (MF-DXA) [12] have shown to be powerful tools in analyzing multifractal behavior of non-stationary time series. The stock market index is a good indicator of the overall market behavior and is frequently used by financial investors. The scale-invariant behavior of both the distribution of price changes and the long term correlations in the absolute values of price changes is a well known property of financial markets [14,15]. Similar behavior was observed for share volume [16,17], and

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it was also found that magnitudes of volume changes display long-term correlations and power-law cross-correlations with magnitudes of price changes [18]. Most work on the multifractality of stock market indices have focused on correlations in price changes [19–21]. However, a more thorough study on the multifractality of price and volume changes across many stock market indices is still lacking.

In this paper we study the multifractal properties of daily price changes and daily volume changes for 13 stock market indices. We apply the MF-DFA method to examine correlations in price changes and volume changes, and look for common behavior across all market indices. We also use the MF-DXA method to analyze cross-correlations between the two market variables, which may provide additional insight into the market's dynamics. The paper is organized as follows. Section 2 introduces a brief overview of the MF-DFA and MF-DXA methods. Section 3 describes our dataset and presents the empirical results. Section 4 draws a conclusion on our analysis.

2. Methodology

2.1. Multifractal detrended fluctuation analysis

Multifractal processes are characterized by different scaling behavior of segments with small and large fluctuations, and their description requires a hierarchy of scaling exponents [10]. We employ the MF-DFA method which quantifies multifractality of non-stationary time series [10], and has been applied to physiological signals [22], geophysical [23], hydrological [24] and financial time series [25,26]. The MF-DFA procedure is briefly described as follows [10]: (i) Integrate the original time series $x(i),\ i = 1, \dots, N$ to produce $X(k) = \sum_{i=1}^{k} [x(i) - \langle x \rangle]$, where $\langle x \rangle$ is the average. (ii) Divide the integrated series $X(k)$ into N_n non-overlapping segments of equal length n and estimate the local trend $X_i(k)$ in each segment from a mth order polynomial regression. (iii) Detrend the integrated series in each segment (by subtracting the local trend) to calculate the detrended variance and average over all segments to find the qth order fluctuation function

$$F_q(n) = \left\{ \frac{1}{N_n} \sum_{i=1}^{N_n} \left[\frac{1}{n} \sum_{k=(i-1)\,n+1}^{in} [X(k) - X_i(k)]^2 \right]^{q/2} \right\}^{1/q} \tag{1}$$

where q can take any real value except zero. (iv) Repeat this calculation to find the fluctuation function $F_q(n)$ for all box sizes n. If long-term correlations are present, $F_q(n)$ will increase with n as a power law $F_q(n) \sim n^{h(q)}$, where the scaling exponent $h(q)$ is calculated as the slope of the linear regression of $\log F_q(n)$ versus $\log n$. Since the scaling exponent $h(2)$ is identical to the well-known Hurst exponent in a stationary time series, $h(q)$ is called the generalized Hurst exponent.

For multifractal processes $h(q)$ is a decreasing function of q and describes scaling behavior of large (small) fluctuations for positive (negative) values of q. These exponents are directly related to the classical multifractal exponents $\tau(q)$ defined in the standard partition function multifractal formalism: $\tau(q) = qh(q) - 1$, where $\tau(q)$ is a linear function for monofractal signals and a nonlinear function for multifractal signals. A multifractal series can also be described by the singularity spectrum $f(\alpha)$ from the Legendre transform $\alpha(q) = \mathrm{d}\tau(q)/\mathrm{d}q, f(\alpha) = q\alpha - \tau(q)$, where $f(\alpha)$ denotes the fractal dimension of the series' subset that is characterized by the Hölder exponent α [10].

In order to measure the complexity of the series, we fit the singularity spectra to a fourth-degree polynomial

$$f(\alpha) = A + B(\alpha - \alpha_0) + C(\alpha - \alpha_0)^2 + D(\alpha - \alpha_0)^3 + E(\alpha - \alpha_0)^4 \tag{2}$$

and calculate a set of multifractal spectrum parameters: the position of maximum α_0; the width of the spectrum $W = \alpha_{max} - \alpha_{min}$ obtained from extrapolating the fitted curve to zero; and the skew parameter $r = (\alpha_{max} - \alpha_0) / (\alpha_0 - \alpha_{min})$ where $r = 1$ for symmetric shapes, $r > 1$ for right-skewed shapes, and $r < 1$ for left-skewed shapes. Roughly speaking, a small value of α_0 suggests the series is correlated and more regular in appearance. The width W measures the degree of multifractality in the series, a wider range of Hölder exponents α results in a "richer" structure and a higher degree of multifractality. The skew parameter r indicates that the scaling behavior of small fluctuations dominates the multifractal behavior if the spectrum is right-skewed, and the scaling behavior of large fluctuations dominates if the spectrum is left-skewed. These three parameters (α_0, W, r) lead to a measure of complexity where a series with a high value of α_0, a wide range W of scaling exponents, and a right-skewed shape can be considered more complex than one with the opposite characteristics [27].

2.2. Multifractal detrended cross-correlation analysis

We use the multifractal detrended cross-correlation analysis (MF-DXA) [12] to analyze the relationship between daily changes in price and volume. This method is a generalization of the MF-DFA and is designed to measure long-term cross-correlations between two simultaneously recorded non-stationary time series. It has been successfully applied in climatic [28], geophysical [29] and financial [30] data. The MF-DXA procedure follows that of MF-DFA in steps (i) and (ii): integrate the time series $x(i), y(i),\ i = 1, \dots, N$ to produce $X(k) = \sum_{i=1}^{k} [x(i) - \langle x \rangle], Y(k) = \sum_{i=1}^{k} [y(i) - \langle y \rangle]$ and calculate the local trends $X_i(k), Y_i(k)$ in each segment from an mth order polynomial regression. (iii) Detrend the integrated series in