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Reducing differences between profiles of weights: A "peer-restricted" cross-efficiency evaluation

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ABSTRACT

This paper deals with the selection of the profiles of weights to be used in cross-efficiency evaluations. In an attempt to prevent unrealistic weighting schemes, one of the issues of main interest that we address here is that of the zero weights, since their use implies that some of the variables considered are excluded from the assessments to be made. In the calculation of cross-efficiency scores, we propose to ignore the profiles of weights of the DMUs that cannot make a choice of non-zero weights among their alternate optima. The different units are therefore assessed in a peer-evaluation that does not consider the profiles of weights of some inefficient DMUs. This approach is referred to as "peer-restricted" cross-efficiency evaluation. Aside from avoiding zero weights, the choice of weights that we make also seeks to reduce the differences between the weights profiles selected as much as possible. Thus, in the "peer-restricted" cross-efficiency evaluation in the present paper we also try to avoid that the different DMUs attach very different weights to the same variable. Finally, we extend this approach to derive a common set of weights by exploiting the idea of similarity between profiles of weights.

1. Introduction

Cross-efficiency evaluation, as proposed in Sexton et al. [1] and Doyle and Green [2], is an extension of the DEA basic methodology [3] aimed at providing a ranking of DMUs. It has also been claimed in the literature that it eliminates unrealistic weighting schemes without the need to elicit weights restrictions. The idea of cross-efficiency evaluation is to assess each unit with the weights of all the DMUs instead of with only its own weights. Unlike the DEA self-evaluation, this provides a peer-evaluation of the DMUs, which makes it possible to derive an ordering. In addition, since the cross-efficiency score of a given unit is usually calculated as the average of the efficiency scores (the cross-efficiencies) obtained with the profiles of weights provided by all the DMUs, it is claimed that the effects of the unrealistic weighting schemes are canceled out in the summary that the cross-efficiency evaluation makes (see also Anderson et al. [4] for discussions). Cross-efficiency evaluation has been used in different contexts: see Sexton et al. [1] for an application to nursing homes, Oral et al. [40] to R&D projects, Doyle and Green [5] to higher education, Green et al. [6] to preference voting, Chen [7] to electricity distribution sector, Lu and Lo [8] to economic–environmental performance and Wu et al. [9,10] to sport at the Summer Olympic.

The main difficulty with cross-efficiency evaluation is the possible existence of alternate optima for the weights when solving the CCR model, which may lead to different cross-efficiency scores depending on the choice of the profile of weights that each DMU makes. The use of alternative secondary goals to the choice of weights among the alternative optimal solutions has been suggested as a potential remedy to the possible influence of this difficulty. In some of the existing proposals along this line each DMU selects an optimal solution for its weights by imposing some condition on the cross-efficiencies of all the DMUs. That is the case of the well-known benevolent and aggressive formulations [1,2]. The benevolent formulation selects weights that maintain the self-efficiency score of the unit under assessment while enhancing the cross-efficiencies of the other DMUs as much as possible, whereas the aggressive formulation also maintains the self-efficiency score while diminishing the rest of cross-efficiencies (see Liang et al. [11] and Wang and Chin [12] for extensions of these models). A different approach to the choice of weights among alternate optima can be found in Ramón et al. [13], where the weights of each DMU are determined without considering their impact on the other DMUs. To be specific, each DMU makes its choice of weights trying to avoid large differences in the weights that each DMU attaches both to the different inputs and to the different outputs. This approach also guarantees non-zero weights.

In this paper, we are particularly concerned with zero weights. The problem with the zeros in cross-efficiency evaluations actually comes from DEA. The DEA total flexibility in the choice of

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weights often leads to unreasonable results in the sense that the weights provided are frequently inconsistent with the prior knowledge or accepted views on the involved production process. In particular, the problem with the zero weights has been pointed out: DEA often assesses DMUs putting the weight only on a few set of inputs and outputs, ignoring the remaining variables by assigning them a zero weight. This is why the literature has widely claimed the need to avoid zero weights in efficiency assessments. To prevent zero weights in cross-efficiency evaluations and, therefore, avoiding that some of the variables considered are excluded from the assessments, we propose to ignore the profiles of the DMUs that have zeros in all their alternative optimal solutions for the weights, as well as to make a suitable choice of non-zero weights among the alternate optima in the case of the remaining DMUs. We have called this approach "peer-restricted" cross-efficiency evaluation, which can be seen as an intermediate approach in between DEA, which provides a self-evaluation of each unit, and the standard cross-efficiency evaluation, which assesses each unit with the profiles of weights of all the DMUs. Ignoring the weights profiles of some DMUs in the cross-efficiency evaluation has already been done in the literature. See Lam [14], where only the weights of the efficient DMUs are used, since the author states that only these DMUs are expected to have relative strengths in some inputs and outputs. We finally note that, to prevent from using the weights profiles provided by the CCR model for the inefficient DMUs that are assessed with slacks (and so, they have zero weights) in cross-efficiency evaluations, it has also been proposed to use those resulting from a re-assessment of these DMUs by means of some suitable weights (see Ramón et al. [13]).

As for the choice of weights that we make among the alternate optima in the CCR model, we not only look for non-zero weights but also we seek that such choice is made by reducing as much as possible the differences between the profiles of weights selected. Reducing differences between profiles of weights means to reduce differences in the weights that the different DMUs attach to the same variable. The DEA general literature has claimed the need to exercise some control over the variation in factor weights resulting from the DEA flexibility (see Roll and Golany [15] for discussions). In particular, Roll et al. [16] state that "In some cases (and for certain purposes) it may be considered unacceptable that the same factor is accorded widely differing weights, when assessing different units"; Pedraja-Chaparro et al. [17] claim that "Although some degree of flexibility on the weights may be desirable for the DMUs to reflect their particular circumstances it may often be unacceptable that the weights should vary substantially from one DMU to another"; and Thanassoulis et al. [18] state that "It may be desirable to reduce the dispersion in the optimal weights assigned to each factor by each DMU. In the extreme case where no flexibility is allowed there is the common set of weights (CSW)". In the context of cross-efficiency evaluation, we also argue that looking for the profiles of weights that are most similar among themselves may be desirable, since these will produce more similar cross-efficiencies (with less dispersion) and, consequently, we will have more representative cross-efficiency scores as these latter are usually calculated as the average of the cross-efficiencies (see Wang and Chin [41] for a different approach that does not utilize the usual average of cross-efficiencies and proposes to aggregate them with weights that are not necessarily equal; these are determined by using the ordered weighted averaging (OWA) operator).

Finally, we extend the proposed approach to derive a common set of weights (CSW). CSW, as first denoted in Roll et al. [16], is an approach different from cross-efficiency evaluation that can also be used for assessing the efficiency of the DMUs and ranking them. It is frequently used in practice; in particular, it may be a suitable approach when there is no need to allow for individual circumstances regarding the conditions of operation of the different DMUs. We here exploit the idea of similarity between profiles of weights and go one step further by proposing a CSW as the average profile of the most similar profiles of weights provided by our approach. Other considerations regarding the summary of these weights profiles are also discussed.

The paper unfolds as follows: in Section 2, we discuss the selection of the profiles of weights to be used in cross-efficiency evaluations. Section 3 addresses the choice of weights by using a criterion of similarity between weights profiles. In a subsection the models proposed are modified in order to guarantee non-zero weights. In Section 4 we extend the proposed approach to derive a common set of weights. Section 5 includes an illustrative example. Finally, Section 6 concludes.

2. Strategies for the selection of profiles of weights: the "peer-restricted" cross-efficiency evaluation

Throughout the paper we assume that we have n DMUs that use m inputs to produce s outputs and that their relative efficiency is assessed with the CCR model. Then for a given DMU_0, we have to solve the following model:

$$\begin{aligned} \text{Max} \quad & \frac{\sum_{r=1}^{s} u_r y_{r0}}{\sum_{i=1}^{m} v_i x_{i0}} \\ \text{s.t:} \quad & \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} \le 1 \quad j=1,\ldots,n \qquad v_i,\ u_r \ge 0 \quad \forall i,r \end{aligned} \tag{1}$$

which can converted into the following linear problem, called the dual multiplier formulation, using the results on linear fractional problems in Charnes and Cooper [19]:

$$\begin{aligned} \text{Max} \quad & \sum_{r=1}^{s} u_r y_{r0} \\ \text{s.t.} \quad & \sum_{i=1}^{m} v_i x_{i0} = 1 \\ & -\sum_{i=1}^{m} v_i x_{ij} + \sum_{r=1}^{s} u_r y_{rj} \le 0 \quad j=1,\ldots,n \\ & v_i, u_r \ge 0 \quad \forall i,r \end{aligned} \tag{2}$$

In the standard cross-efficiency evaluation the optimal solutions of (2) for each DMU_d, $(v_1^d,\ldots,v_m^d,u_1^d,\ldots,u_s^d)$, provide the profiles of weights that are used to calculate the cross-efficiency of a given DMU_j, j=1, …, n, as follows:

$$E_{dj} = \frac{\sum_{r=1}^{s} u_r^d y_{rj}}{\sum_{i=1}^{m} v_i^d x_{ij}} \tag{3}$$

and the cross-efficiency score of DMU_j is defined as the average of the these cross-efficiencies

$$\bar{E}_j = \frac{1}{n}\sum_{d=1}^{n} E_{dj}, \quad j=1,\ldots,n \tag{4}$$

which measures the average efficiency according to all DMUs.

As said before, we are particularly concerned with the zero weights, since the use of profiles of weights with zeros would imply that some of the variables considered are excluded from the assessments. It has been claimed in the literature that cross-efficiency evaluation eliminates unrealistic weighting schemes in the sense that their effects are canceled out in the amalgamation of cross-efficiencies of the different DMUs. However, this cannot obviously be guaranteed. As a simple illustration, in Table 1 we show the profiles of weights obtained with the benevolent formulation for the data in Shang and Sueyoshi [20] (this table has been taken from Ramón et al. [13]). We can see that all the