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Optimal portfolio positioning under ambiguity☆

H. Ben Ameur [a], J.L. Prigent [b,*]

[a] INSEEC Business School, 27 rue claude vellefaux 75010 Paris, France
[b] ThEMA, University of Cergy-Pontoise, 33 Bd du Port, 95011, Cergy-Pontoise, France

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ABSTRACT

This paper analyzes the optimality of financial portfolios when the investor has a utility with ambiguity aversion. It provides a general result about the optimal portfolio profile under ambiguity, in the Anscombe–Aumann framework, using the Maccheroni et al. (2006) approach which includes Gilboa and Schmeidler's (1989) multiple prior preferences and Hansen and Sargent's (2011) multiplier preferences. The paper then details the CRRA case with an ambiguity index based on relative entropy. Such findings have practical applications in structured portfolio management. Indeed, it is important to take account of uncertainty about the true values of financial parameters when determining the best portfolio profile.

1. Introduction

Since the seminal work of von Neumann and Morgenstern (1947), Expected Utility theory (EU) has been widely applied in order to model investors' attitudes towards risk. Markowitz (1952) first determined the optimal static portfolio solution in the mean-variance framework. This approach is related to quadratic utility functions. Merton (1971) determined the continuous-time optimal portfolio for various utility functions. These fundamental results have been further extended, for example by taking account of market incompleteness, specific constraints on portfolio weights, labor income and random horizon…as in Cox and Huang (1989), Cvitanic and Karatzas (1996), and, with insurance constraints, in El Karoui et al. (2005) and Prigent (2006) (see also Campbell and Viceira, 2002; Prigent, 2007, for an overview of such results). However, some well documented paradoxes, such as Allais's paradox, have shown that standard utility theory does not model real attitudes towards risk. Allais (1953) suggests that the independence axiom is not often validated by actual individual behaviors. Cohen and Tallon (2000) also mention that the expected utility theory implies that the utility function must simultaneously both formalize the choice among alternatives and models risk aversion. Therefore, an investor with decreasing marginal utility must necessarily be risk averse.

Among various alternatives to the standard expected utility theory, some authors have suggested that common knowledge of probability distributions is a hypothesis too strong in rational expectations. Under this assumption, all individuals have the same opinion about the "true" probability distribution of random events, which is unique. But, the usual uncertainty about true financial parameter values leads to misspecification. In this context, probability distributions are no longer uniquely determined. To overcome this problem, Hansen and Sargent (2000, 2011) have introduced robust control models. The robust preference approach considers that individual objective functions take account of the possibility that the model used by the individual may be false and is only an approximation of the true model. Hansen and Sargent (2000, 2011) have argued that uncertainty can be based on ambiguity, resulting from the lack of precise information about randomness. Knight (1921) distinguishes between risk and uncertainty. Risk refers to a situation where probabilities are known to guide choice, while uncertainty refers to a situation where information is too vague to define probabilities.

As mentioned previously, we must often take account of ambiguity about probability distributions. The notion of ambiguity was introduced by Ellsberg (1961). Ellsberg (1961) called into question Savage's theory (1954), whereby individual subjective beliefs on the likelihood of possible states are subjective probabilities. Ellsberg (1961) carried out a simple experiment: one urn contains 50 red balls and 50 black balls; a second urn contains a combination of the two but we do not know in which proportions. It was observed that people typically preferred to bet on a ball from the urn with the known mixture than from that where the proportions were unknown. This shows that individual choice under uncertainty depends on the consequences, on the probabilities associated with these consequences and also on the confidence that individuals have in these probabilities. This evidence reflects aversion to ambiguity: individuals prefer to act on known rather than unknown probabilities. Gilboa and Schmeidler (1989) have considered

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* Corresponding author. Tel.: +33 1 34 25 61 72; fax: +33 1 34 25 62 33.
E-mail addresses: hbenameur@groupeinseec.com (H.B. Ameur), jean-luc.prigent@u-cergy.fr (J.L. Prigent).

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the so-called “maxmin expected utility preferences”, which assume the existence of multiple priors. Maccheroni et al. (2006) propose a model in accordance with the standard Anscombe and Aumann (1963) approach, based on specific assumptions about both the utility function and the ambiguity index. This model includes the case of multiple-prior preferences considered by Gilboa and Schmeidler (1989), multiplier preferences introduced by Hansen and Sargent (2011), and mean-variance preferences of Markowitz (1952) and Tobin (1958). Asano (2012) focuses on portfolio inertia in the context of Knightian uncertainty. He considers two cases: the first corresponds to preference represented by Choquet's expected utility theory and the second to preference represented by the maxmin expected utility axiomatized by Gilboa and Schmeidler (1989). Asano (2012) analyzes the effect of uncertainty on the spread between buying and selling prices in stock markets. Qu (2011) proposes a generalization of the maxmin expected utility model and of the subjective expected utility model. In this framework, ambiguity and unambiguity are distinguished through individuals' different representations. Taking the Knightian distinction into account, Qu (2011) suggests a subjective definition of ambiguity, especially in the context of biseparable preference.

Portfolio optimization under ambiguity has been examined in various frameworks. For standard portfolio allocation as introduced by Markowitz (1952), the investor must choose his portfolio weights at the initial date. Portfolio returns are linear combinations of asset returns. In this case, Pflug and Wozabal (2007) use a maximin criterion based on a confidence set for probability distribution. They illustrate the trade-off between return, risk and robustness with respect to ambiguity and provide a monetary valuation of the information (see also Wozabal (2012) for the case of non-parametric ambiguity sets). Calafiore (2007) determines optimal robust portfolios when assuming that a nominal discrete return distribution is given, while the true distribution is unknown except that it lies within a given distance from the nominal one computed according to the Kullback–Leibler divergence criterion. Calafiore (2007) determines portfolios that minimize the maximum among all the allowable distributions of a given weighted risk-mean objective (in particular, the measures of standard variance and absolute deviation). Additionally, Pflug et al. (2012) show that uniform investment strategy (i.e. the equally weighting 1/N investment strategy) is rational for investors facing a significantly high degree of ambiguity about loss distributions, for a large class of risk measures. Koziol et al. (2011) deal with the ambiguity of institutional investors towards specific assets. By estimating the average portfolio weights for standard and alternative asset classes of 119 institutional investors, the model can be calibrated to identify the ambiguity factors of each asset type. These authors show that institutional investors are strongly ambiguity-averse and that equities and bonds have much lower ambiguity than other investments such as real estate, private equities, and hedge funds. In the continuous-time framework, Fei (2007) examines the optimal portfolio choice with respect to the recursive multiple-prior utility. He explicitly provides the optimal consumption and portfolio values for power and logarithmic felicity functions. Liu (2011) examines the same problem where expected returns of a risky asset follow a hidden Markov chain. He proves that ambiguity aversion emphasizes the importance of hedging demands in optimal portfolio strategies.

In this paper, we provide the optimal portfolio payoff under ambiguity, in the optimal portfolio positioning framework introduced by Leland (1980), and Brennan and Solanki (1981). Note that portfolio positioning refers to static strategies. But actual portfolio hedging strategies in fact correspond to discrete-time trading. Additionally, structured portfolio management is based particularly on initial positioning on financial derivatives. The portfolio value is a function of a given benchmark. The portfolio payoff maximizes the investor's expected utility. When taking account of the ambiguity index, the investor's risk aversion and ambiguity index characterize the optimal portfolio profile, which involves option-based strategies. A particular case of optimal positioning is *Option Based Portfolio Insurance* (OBPI), introduced by Leland and Rubinstein (1976). This consists of a portfolio invested in a risky asset S (usually a financial index such as the *S*&*P*) covered by a listed put written on it. Whatever the value of S at given horizon T is, the portfolio value is always above the strike K of the put. Portfolio insurance theory usually considers portfolio payoffs which are functions of a benchmark (a specified portfolio of common assets). At maturity, downside risk is limited (investors can receive a given percentage of their initial capital, even in bearish markets), while investors may also participate in upside markets. However, more specific insurance constraints can be introduced, for example for institutional investors (see e.g. Bertrand et al. (2001) for quite general insurance constraints). Our purpose is to analyze how the optimal portfolio profile is modified when taking account of ambiguity. In practice, ambiguity is typically due to uncertainty about the true values of financial parameters. Determining the optimal portfolio in such a framework therefore has important applications for the structured portfolio management industry.

The paper is organized as follows. Section 2 presents an overview of ambiguity theory, mainly the Maccheroni et al. (2006) approach. Section 3 provides the general result of portfolio positioning under ambiguity aversion. Within this framework, Section 4 illustrates the general result by examining a fundamental example that emphasizes the role of aversions to both risk and ambiguity.

2. The concept of ambiguity

Two main notions of ambiguity have been proposed. Epstein (1999) considers ambiguity neutrality with respect to probabilistically sophisticated preferences. Ghirardato and Marinacci (2002) identify ambiguity neutrality with subjective expected utility preferences. These authors consider subjective expected utility preferences as ambiguity neutral preferences. The notion of ambiguity aversion provides a foundation for standard exercises of comparative statics in ambiguity for multiple prior preferences that are based on the size of the set of priors. This is the case of Hansen and Sargent (2011) multiplier preferences, which are easily seen to be probabilistically sophisticated. Consider an individual who has to make choices and faces limited information about what may happen. Usually, this individual will be cautious.[1] Remaining in the Von Neumann–Morgenstern formalization and particularly in Savage's model, Gilboa and Schmeidler (1989) propose accommodating ambiguity within economic decision making and assume that in the presence of ambiguity, individuals cannot identify a single probability distribution over states of nature. They thus consider multiple probability distributions and then evaluate their choices according to the worst probability distribution for that choice. This is the approach based on multiple priors. The decision model is called “maxmin expected utility” (MEU). This model is flexible, allows a distinction between risk and ambiguity and can capture preference with ambiguity aversion. Maccheroni et al. (2006) characterize the preferences under ambiguity by introducing both utility function U on outcomes and an ambiguity index C on the set of probabilities defined on the random events. Thus, they consider the following representation of preferences:

For all random variables X and Y which represent results or consequences and with values in $[-M,M]$, we have:

$$\begin{array}{ccc} X & \succeq & Y \Longleftrightarrow \\ Min_{\mathbb{P}\in\Delta}\int U(X)d\mathbb{P}+C(\mathbb{P}) & \geq & Min_{\mathbb{P}\in\Delta}\int U(Y)d\mathbb{P}+C(\mathbb{P}). \end{array} \tag{1}$$

The function U corresponds to decision risk attitude. Index C represents individual attitude towards ambiguity. This representation of preferences includes both the multiple prior preferences of Gilboa and Schmeidler (1989) and the multiplier preferences of Hansen and

[1] This attitude can be relevant in many cases including Ellsberg-type choice situations.