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Optimal portfolio choice for an insurer with loss aversion

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ABSTRACT

The problem of optimal investment for an insurance company attracts more attention in recent years. In general, the investment decision maker of the insurance company is assumed to be rational and risk averse. This is inconsistent with non fully rational decision-making way in the real world. In this paper we investigate an optimal portfolio selection problem for the insurer. The investment decision maker is assumed to be loss averse. The surplus process of the insurer is modeled by a Lévy process. The insurer aims to maximize the expected utility when terminal wealth exceeds his aspiration level. With the help of martingale method, we translate the dynamic maximization problem into an equivalent static optimization problem. By solving the static optimization problem, we derive explicit expressions of the optimal portfolio and the optimal wealth process.

1. Introduction

Traditional portfolio choice theory is generally based on the model of expected utility maximization (EUM). The model is premised upon the assumption that decision makers are rational and risk averse when facing uncertainty. This assumption, however, has long been challenged by many empirical results and market anomalies such as the Allais paradox, the return reversals and the equity premium puzzle.

In recent years, a number of theories have been proposed to remedy the drawbacks of EUM, such as Lopes' SP/A model (Lopes, 1987), and Kahneman and Tversky's Cumulative Prospect Theory (CPT; Tversky and Kahneman, 1992). Certainly the most famous among them is the Cumulative Prospect Theory (CPT). So more and more researchers take more interest in incorporating it into the optimal portfolio selection problem. The early literature considered is a static portfolio selection problem, for example Benartzi and Thaler (1995), Lopes and Oden (1999), Shefrin and Statman (2000), Enrico and Thierry (2008) and Bernard and Ghossoub (2010). Berkelaar et al. (2004) earlier study the dynamical portfolio selection problem under CPT. They consider a very specific two-piece power utility function and derive the optimal investment strategy of loss averse investors. Jin et al. (2008) formulated a continuous-time portfolio selection model under CPT. To solve the optimization problem they develop a Choquet maximization and minimization technique. Zhang et al. (2011) investigate a same CPT portfolio selection model where the loss is a priori bounded by a given level. Mi and Zhang (2012) also investigate an optimal portfolio selection model under CPT. However, they consider the market setting is incomplete.

In addition, with permission of the insurance companies to invest in capital market in practice, the problem of optimal investment for a general insurance company has attracted more and more attention. Generally, the investment decision makers are assumed as rational and risk averse investors, such as Browne (1995), Hipp and Plum (2000, 2003), Paulsen and Gjessing (1997), Liu and Yang (2004), Yang and Zhang (2005) and Cai and Xu (2006). But in reality, as the personality of the people, the investment decision makers of insurance company are not always fully rational. To the best of our knowledge there has been no report incorporating CPT into the optimal insurance investment problem. In this paper, we consider a continuous-time optimal portfolio selection model under loss aversion. The decision maker's preference in facing market risks is defined by an S-shaped utility function featuring the reference point. In order to make our model more practical, we consider a financial market of one riskless asset and n risk assets and the surplus process of the insurance company is modeled by a Lévy process. Furthermore, by the help of the traditional martingale method, we derive the closed-form optimal portfolio and the optimal wealth process.

The organization of this paper is as follows. In Section 2, the problem is formulated. The explicit expression of the optimal portfolio and the optimal wealth process are worked out by a martingale approach in Section 3.

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2. Formulation of the problem

We assume that the insurance company may invest in capital market where $n+1$ assets are traded continuously on a finite horizon $[0, T]$. One is a riskless asset with price $S_0(t)$ given by

$$dS_0(t) = S_0(t)r(t)dt, \quad S_0(0) = 1 \tag{2.1}$$

and n risk assets with prices $S_i(t)$ satisfying

$$dS_i(t) = S_i(t)\left[b_i(t)dt + \sum_{j=1}^{n} \sigma_{ij}(t)dB_j(t)\right],$$
$$S_i(0) > 0,\ j = 1, 2, \ldots, n \tag{2.2}$$

where $r(t)$ is the interest rate and $B(t) := (B_1(t), \ldots, B_n(t))^T$ is an n-dimensional Brownian motion on a probability space $(\Omega^B, \mathcal{F}^B, \mathbb{P}^B)$ with the component Brownian motion $B_j(t), j = 1, \ldots, n$, being independent. The drift coefficients vector $b(t) := (b_1(t), \ldots, b_n(t))^T$ and volatility matrix $\sigma(t) := \{\sigma_{ij}(t)\}_{n\times n}$ are assumed to be deterministic and bounded uniformly in $[0, T]$. In addition, we assume that the matrix $\sigma(t)$ satisfies the nondegeneracy condition

$$\sigma(t)\sigma(t)^T \geq \delta I, \quad \forall t \in [0, T], \tag{2.3}$$

where $\delta > 0$ is a given constant.

The surplus process $U(t)$ of an insurer is assumed to be the compound Poisson risk process or the classical Cramér–Lundberg model, namely

$$U(t) = x + \int_0^t \alpha(s)ds - S(t) = x + \int_0^t \alpha(s)ds - \sum_{i=1}^{K(t)} Z_i, \tag{2.4}$$

where $x > 0$ is the initial reserve of an insurance company; $\alpha(t) > 0$ is the premium rate at time t; $S(t) = \sum_{i=1}^{K(t)} Z_i$ is a compound Poisson process defined on probability space $\{\Omega^S, \mathcal{F}^S, \mathbb{P}^S\}$, $\{\mathcal{F}_t^S\}$ is the $\mathbb{P}^S$-augmentation of natural filtration $\sigma(S(s); 0 \leq s \leq t)$, and $K(t)$ is a homogeneous Poisson process with intensity $\lambda(t)$ and represents the number of claims occurring in time interval $[0, t]$; and $\{Z_i\}_{i=1}^{\infty}$ is an i.i.d. sequence of non-negative random variables, modeling the incoming claim sizes, the common distribution function of $\{Z_1, Z_2, \ldots\}$ is denoted by $F(z) = Pr\{Z \leq z\}$, namely, $\{Z_1, Z_2, \ldots\}$ have the same distribution as Z. We assume $E(Z) = \int_0^\infty zdF(z) < \infty$ and $E(Z^2) = \int_0^\infty z^2dF(z) < \infty$. Let

$$(\Omega, \mathcal{F}, \mathbb{P}) = \left(\Omega^B \times \Omega^S,\ \mathcal{F}^B \otimes \mathcal{F}^S,\ \mathbb{P}^B \otimes \mathbb{P}^S\right) \tag{2.5}$$

denote the product space. In this space, $B(t)$, $K(t)$, $\{Z_i\}_{i=1}^{\infty}$ are mutually independent.

Let $L(t)$ denote the compensated compound Poisson process, i.e.

$$L(t) := S(t) - E(Z)\int_0^t \lambda(s)ds.$$

Then $L(t)$ is a 1-dimensional compensated pure Lévy process defined in $(\Omega, \mathcal{F}, \mathbb{P})$ and is a martingale (see, e.g., Shreve, 2004). Moreover $B(t)$ and $L(t)$ in $(\Omega, \mathcal{F}, \mathbb{P})$ are mutually independent.

Let N denote the Poisson random measure (or jump measure) of L and ν denote the Lévy measure that satisfies

$$\nu(0) = 0, \quad \int_{\mathbb{R}} (1 + |x|^2)\nu(dx) < \infty. \tag{2.6}$$

Intuitively speaking, the Lévy measure describes the expected number of jumps of a certain height in a time interval of length 1. For the risk process model (2.4), we have $\nu(dx) = \lambda F(dx)$.

According to Øksendal and Sulem (2005), L has the following Lévy decomposition:

$$L(t) = \int_0^t \int_{\mathbb{R}} z\left(N(ds, dz) - \nu(dz)ds\right). \tag{2.7}$$

Then the surplus process (2.4) can be re-written as

$$U(t) = x + \int_0^t c(s)ds - L(t),$$

where $c(t) = \alpha(t) - \lambda(t)E(Z)$.

The insurer is allowed to choose a portfolio consisting of n risky assets and a riskless asset. We define $\pi(t) = (\pi_1(t), \ldots, \pi_n(t))^t$ as the portfolio of the insurer with $\pi_i(t)$, denoting the amount invested in the ith stock at time t for $i = 1, 2, \ldots, n$.

Definition 2.1. A portfolio process $\pi(t) = (\pi_1(t), \ldots, \pi_n(t))^T$ is said to be admissible if $\pi(t)$ is an $\mathcal{F}_s$-predictable process such that

$$\int_0^T \|\pi(t)\|^2 dt < \infty, \quad \text{a.s. for all } T < \infty. \tag{2.8}$$

The set of all admissible portfolios is denoted by Π.

The insurer would invest in the $n+1$ assets continuously. We denote his wealth at time t by $X(t)$. Then $X(t)$ satisfies the following stochastic differential equation:

$$\begin{cases} dX(t) = \left(X(t)r(t) + \pi(t)^T(b(t) - r(t)\mathbf{1_n}) + c(t)\right)dt \\ \quad + \pi(t)^T\sigma(t)dB(t) - dL(t), \\ X(0) = x, \end{cases} \tag{2.9}$$

where $\mathbf{1_n}$ denotes n-dimensional column vector with every entry equal to 1.

In the conditional optimal portfolio theory for the insurer employing expected utility maximization, the utility function is assumed smooth, concave and nondecreasing in $(0, \infty)$. In this paper, we consider an S-shaped utility function in the CPT of Tversky and Kahneman (1992), i.e. the insurer is assumed to be an investor with loss aversion.

Define

$$u(x) = \begin{cases} u_1(x), & x > 0; \\ u_2(x), & x \leq 0; \end{cases} \tag{2.10}$$

where $u_1(\cdot) : R^+ \mapsto R^+$, is strictly increasing, concave and twice differentiable with $u_1'(0) = 0, u_1'(0+) = +\infty$ and $u_1'(+\infty) = 0$ (Inada conditions[1]); $u_2(\cdot) : R^- \mapsto R^-$, is strictly increasing, convex and twice differentiable with $u_2(0) = 0, u_2'(0-) = +\infty$ and $u_2'(-\infty) = 0$. Moreover, $u_2(\cdot)$ is steeper than $u_1(\cdot)$ which reflects that the investor is more sensitive to losses than gains.

A particular utility function

$$u(x) = \begin{cases} \alpha x^{\gamma_1}, & x > \xi; \\ -\beta(-x)^{\gamma_2}, & x \leq \xi; \end{cases} \tag{2.11}$$

where $0 < \gamma_1 \leq \gamma_2 \leq 1, \beta > \alpha > 0, \beta$ is the loss aversion coefficient of the investor with $\gamma_1 = \gamma_2 = 0.88, \alpha = 1, \beta = 2.25$ as a special case, suggested by Tversky and Kahneman (1992) will be applied in this paper.

The utility function $U(\cdot)$ of the insurer can be described as

$$U(X(t)) = u_1(X(t) - \xi)\mathbf{I}_{X>\xi} + u_2(X(t) - \xi)\mathbf{I}_{X\leq\xi} \tag{2.12}$$

where $\xi > 0$ is a constant representing the reference point that determines whether a terminal wealth profile is a gain or a loss. $\mathbf{I}$ is an indicator function.

Following utility maximization criterion, the problem of choosing an optimal portfolio for an insurer can be formulated as follows:

$$\begin{cases} \max_{\pi\in\Pi} U(X(T)) \\ \text{s.t. } X(t) \quad \text{satisfies (2.9)}, \\ X(t) \geq 0, \quad \forall t \in [0, T] \end{cases} \tag{2.13}$$

where $X(t) \geq 0, \forall t \in [0, T]$ reflects that the insurance company is not bankrupt throughout the investment period $[0, T]$.

[1] To know more about Inada conditions, one can refer to Karatzas and Shreve (1998).