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Stochastic lot sizing problem with controllable processing times ☆

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ABSTRACT

In this study, we consider the stochastic capacitated lot sizing problem with controllable processing times where processing times can be reduced in return for extra compression cost. We assume that the compression cost function is a convex function as it may reflect increasing marginal costs of larger reductions and may be more appropriate when the resource life, energy consumption or carbon emission are taken into consideration. We consider this problem under static uncertainty strategy and α service level constraints. We first introduce a nonlinear mixed integer programming formulation of the problem, and use the recent advances in second order cone programming to strengthen it and then solve by a commercial solver. Our computational experiments show that taking the processing times as constant may lead to more costly production plans, and the value of controllable processing times becomes more evident for a stochastic environment with a limited capacity. Moreover, we observe that controllable processing times increase the solution flexibility and provide a better solution in most of the problem instances, although the largest improvements are obtained when setup costs are high and the system has medium sized capacities.

1. Introduction

In this paper, we consider the lot sizing problem with controllable processing times where demand follows a stochastic process and processing times of jobs can be controlled in return for extra cost (compression cost). Processing time of a job can be controlled (and reduced) by changing the machine speed, allocating extra manpower, subcontracting, overloading, consuming additional money or energy. Although these options are available in many real life production and inventory systems, in the traditional studies on the lot sizing problem, processing times of jobs are assumed as constant.

Since the seminal paper of Wagner and Whitin [40], the lot sizing problem and its extensions have been studied widely in the literature (see [13,23] for a detailed review on the variants of the lot sizing problem). In the classical lot sizing problem, it is assumed that the demand of each period is known with certainty although this is not the case for most of the production and inventory systems and approximating the demand precisely may be very difficult. In the stochastic lot sizing problem, this assumption is relaxed but the probability distribution of the demand is assumed as known.

As reducing processing time of a job is equivalent to increasing production capacity, subcontracting, overloading or capacity acquisition can be seen as special cases of the controllable processing times. There are studies in the literature that consider the lot sizing problem with subcontracting (or outsourcing) [3,10,18] or capacity acquisition (or expansion) [1,17,22]. However, in all these studies costs of these options are assumed as linear or concave. This assumption makes it possible to extend the classical extreme point or optimal solution properties for these cases. In our study, we assume that the compression cost is a convex function of the compression amount.

Controllable processing times are well studied in the context of scheduling. Earlier studies on this subject assume linear compression costs as adding nonlinear terms to the objective (total cost) function may make the problem more difficult [14]. However, as it is stated in recent studies, reducing processing times gets harder (and more expensive) as the compression amount increases in many applications [14,2]. For example, by increasing machine speed, processing times can be reduced, but this also decreases life of the tool and an additional tooling cost is incurred. Moreover, increasing the machine speed may also increase the energy consumption of the facility. Another example is a transportation system in which trucks may be overloaded or their speeds could be increased in return for extra cost due to increasing fuel consumption or limiting the carbon emission. Thus, considering a convex compression cost function is realistic since a convex function represents increasing marginal costs and may limit higher usage of the resource due to environmental issues.

In our study, we consider the following convex compression cost function for period t: $\gamma_t(k_t)=\kappa_t(k_t)^{a/b}$ where $k_t>0$ is the total compression amount in period t, $\kappa_t\ge 0$ and $a\ge b>0$, $a,b\in Z_+$. Note that, for $a>b$ and $\kappa_t>0$, γ_t is strictly convex. This function can

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represent increasing marginal cost of compressing processing times in larger amounts. Moreover, this function can be related to a (convex) resource consumption function [25,28]. Suppose that one additional unit of the resource costs κ_t and for compressing the processing time by k_t units, additional $k_t^{a/b}$ units of resource should be allocated. Thus, in this context, compression cost represents resource consumption cost and the resource may be a continuous nonrenewable resource such as energy, fuel or catalyzer. With the recent advances in convex programming techniques, many commercial solvers (like IBM ILOG CPLEX) can now solve second-order cone programs (SOCP). In this study, we make use of this technique and formulate the problem as SOCP so that it can be solved by a commercial solver.

The contributions of this paper are threefold:

- To the best of our knowledge, this is the first study that considers the stochastic lot sizing problem with controllable processing times. Although this option is applicable to many real life systems, the processing times are assumed as constant in the existing literature on lot sizing problems.
- The inclusion of a nonlinear compression cost function complicates the problem formulation significantly. Therefore, we utilize the recent advances in second-order cone programming to alleviate this difficulty, so that the proposed conic formulations could be solved by a commercial solver in a reasonable computation time instead of relying on a heuristic approach.
- Since assuming fixed processing times unnecessarily limits the solution flexibility, we conduct an extensive computational experiments to identify the situations where controlling the processing times improves the overall production cost substantially.

The rest of the paper is organized as follows. In the next section, we briefly review the related literature. In Section 3, we formulate the problem and in Section 4, we strengthen the formulation using the second-order conic strengthening. In Section 5, we present the results of our computational experiments. We first compare alternative conic formulations presented in Section 5, afterwards we investigate the impact of controllable processing times on production costs. In Section 6, conclusions and future research directions are discussed.

2. Literature review

Here, we first review the studies on stochastic lot sizing problems. Silver [30] suggests a heuristic solution procedure for solving the stochastic lot sizing problem. Laserre et al. [16] consider the stochastic capacitated lot sizing problem with inventory bounds and chance constraints on inventory. They show that solving this problem is equivalent to solving a deterministic lot sizing problem. Bookbinder and Tan [5] study the stochastic uncapacitated lot sizing problem with α-service level constraints under three different strategies (static uncertainty, dynamic uncertainty and static-dynamic uncertainty). Service level α represents the probability that inventory will not be negative. In other words, it means that with probability α, the demand of any period will be satisfied on time. Under the static uncertainty decision rule, which is the strategy that will be used in our study, all the decisions (production and inventory decisions) are taken at the beginning of the planning horizon (frozen schedule). The authors formulate the problem and show that their model is equivalent to the deterministic problem by showing the correspondence between the terms of these two formulations.

Service level constraints are mostly used in place of shortage or backlogging costs in the stochastic lot sizing problems. Since shortages may lead to loss of customer goodwill or delays on the other parts of the system, it may be hard to estimate the backlogging or shortage costs in the real life production and inventory systems. Rather than considering the backlogging cost as a part of the total cost function, a specified level of service (in terms of availability of stock) can be assured by service level constraints and when the desired service level is high, backlogging costs can be omitted. This situation makes the usage of service level constraints more popular in the real life systems [5,19,6]. A detailed investigation of different service level constraints can be found in Chen and Krass [6].

Vargas [38] studies (the uncapacitated version of) the problem of Bookbinder and Tan [5] but rather than using service level constraints he assumes that there is a penalty cost for backlogging, the cost components are time varying and there is a fixed lead time. He develops a stochastic dynamic programming algorithm, which is tractable when the demand follows a normal distribution. Sox [31] studies the uncapacitated lot sizing problem with random demand and non-stationary costs. He assumes that the distribution of demand is known for each period and considers the static-uncertainty model, but uses penalty costs instead of service level constraints. He formulates the problem as an MIP with nonlinear objective (cost) function and develops an algorithm that resembles the Wagner–Whitin algorithm.

In the static-dynamic uncertainty strategy of Bookbinder and Tan [5], the replenishment periods are determined first, and then replenishment amounts are decided at the beginning of these periods. They also suggest a heuristic two-stage solution method for solving this problem. Tarím and Kingsman [32] consider the same problem and formulate it as MIP. Moreover, Özen et al. [20] develop a non-polynomial dynamic programming algorithm to solve the same problem. Recently, Tunç et al. [36] reformulate the problem as MIP by using alternative decision variables and Rossi et al. [24] propose an MIP formulation based on the piecewise linear approximation of the total cost function, for different variants of this problem.

In the dynamic uncertainty strategy, production decision for any period is made at the beginning of that period. Dynamic and static-dynamic strategies are criticized due to the system nervousness they cause; supply chain coordination may be problematic under these strategies since the production decision for each period is not known until the beginning of the period [34,35].

There are studies in the literature, in which instead of α service level, fill rate criterion (β service level) is used. Fill rate can be defined as the proportion of demand that is filled from available stock on hand. Thus, this measure also includes information about the backordering size. Tempelmeier [33] proposed a heuristic approach to solve the multi-item capacitated stochastic lot-sizing problem under fill rate constraint. Helber et al. [10] consider the multi-item stochastic capacitated lot sizing problem under a new service level measure, called as δ-service-level. This service level reflects both the size of the backorders and waiting time of the customers and can be defined as the expected percentage of the maximum possible demand-weighted waiting time that a customer is protected against. The authors assume that the cost components are time invariant and there is an overtime choice with linear costs for each period. They develop a nonlinear model and approximate it by two different linear models.

There are also studies in the literature that consider the lot sizing problem with production rate decisions [41] or with quadratic quality loss functions [12]. However, they consider the problem under an infinite horizon assumption.

Another topic related to our study is controllable processing times, which is well studied in the context of scheduling. One of the earliest studies on scheduling with controllable processing times is conducted by Vickson [39]. Kayan and Aktürk [14] and Aktürk et al. [2] consider a CNC machine scheduling problem with controllable processing times and convex compression costs. Jansen and Mastrolilli [11] develop approximation schemes, Gürel et al. [9] use an anticipative approach to form an initial solution, Türkcan et al. [37]