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Fix-and-optimize and variable neighborhood search approaches for multi-level capacitated lot sizing problems ☆

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ABSTRACT

In this paper, a new fix-and-optimize (FO) approach is proposed for two dynamic multi-level capacitated lot sizing problems (MLCLSP), the MLCLSP without setup carryover and the MLCLSP with setup carryover. Given an MIP model of a lot sizing problem, the approach iteratively solves a series of sub-problems of the model until no better solution can be found. Each sub-problem re-optimizes a subset of binary decision variables determined based on the interrelatedness of binary variables in the constraints of the model, while fixing the values of the other binary variables. Based on the FO, a variable neighbourhood search (VNS) approach for the MLCLSP without setup carryover is also developed, which can further improve the solution obtained by the FO by diversifying the search space. Numerical experiments on benchmark instances show that both our FO and VNS approaches can obtain a better solution for most instances compared with that found by the fix-and-optimize approach proposed by Helber and Sahling (International Journal of Production Economics 2010;123:247–256).

1. Introduction

Production planning is one of the most important decisions for manufacturers. It determines how many units of each component/final product should be produced internally or procured from outside suppliers in each period over a given planning horizon, with the objective to minimize the total cost, while meeting customer demand on time. In most enterprise resource planning (ERP) systems, production plans are made using backward scheduling without considering resource capacity constraints as in material requirements planning (MRP) systems. In such planning systems, it is assumed that the lead time for each production/procurement operation is predefined without taking account of the resource capacity constraints of the operation. This might lead to either an infeasible production plan or a plan with excessive inventory. Recently, so-called advanced planning systems or advanced planning and scheduling systems (APS) have emerged as a powerful tool for supply chain planning that considers resource capacity constraints. However, to our best knowledge, most APS systems use a heuristics algorithm to generate a production plan. The performance of such an algorithm cannot be guaranteed in terms of solution quality. Therefore, there is a strong demand for the development of effective and efficient production planning methods that can be implemented in APS systems.

For a manufacturing system which produces final products from raw materials through components, its production planning problem can be formulated as a dynamic multi-level capacitated lot sizing problem (MLCLSP) if the production process is characterized by substantial setup costs and/or setup times. In the MLCLSP, it is assumed that the demand of each final product is known for each period over a planning horizon of multiple periods. The demand must be satisfied on-time. The relationships between the items (raw materials, components, and final products) in the system are given by its bills of materials (BOM). The production of each item in each period requires a manufacturing resource such as a machine, where each resource has a limited capacity. The production of each item in each period requires the setup of the corresponding resource, which incurs a setup time and a setup cost. The objective of the problem is to determine a production plan over the planning horizon for the system such that its total cost including inventory holding costs and setup costs is minimized. The MLCLSP does not consider any setup carryover between two successive periods. As an extension, the MLCLSP with linked lot sizes or the MLCLSP-L for short allows the setup state of a resource to be carried over from the current period to the next period. Both the MLCLSP and the MLCLSP-L are NP-hard. Their

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high complexity has been recognized through the studies in the past three decades. Up to now, for many benchmark instances proposed by Tempelmeier and Derstroff [1] for the MLCLSP, their integrality gap still remains high. Trigeiro et al. [2] pointed out that for the capacitated lot sizing problem (CLSP) with setup times, its feasibility problem (the problem to check whether it has a feasible solution) is NP-complete. As the MLCLSP and the MLCLSPL can be reduced to the CLSP with setup times, they are at least as hard as the CLSP and hence also *NP*-hard.

In this paper, we study both the MLCLSP and the MLCLSP-L and propose a new fix-and-optimize (FO) approach different from those of Helber and Sahling [3] and Sahling et al. [4]. This is an iterative approach. Given an MIP model of a problem, this approach first constructs an initial feasible solution of the model by simply setting its all binary setup variables to one as in [3,4]. In each iteration of the approach, a binary setup variable of the model is randomly selected and tagged as a binary variable to be re-optimized. At the same time, all binary setup variables related to the selected variable in the model are also tagged as binary variables to be re-optimized, whereas all other binary variables are fixed at their values obtained in the last iteration. This leads to an MIP submodel (subproblem) with fixed binary variables, binary variables to be re-optimized, and real variables (also to be re-optimized). The submodel is solved by using an MIP solver and the current best solution of the original MIP model is replaced by the optimal solution of the submodel if the latter solution is better. This iterative procedure continues until no submodel has a solution better than the current best solution of the MIP model. Based on the fix-and-optimize approach, a variable neighbourhood search (VNS) approach for the MLCLSP without setup carryover is also developed, which can further improve the solution obtained by the FO by diversifying the search space.

Compared with the fix-and-optimize approach proposed by Helber and Sahling [3] for the MLCLSP and that proposed by Sahling et al. [4] for the MLCLSP-L, our FO approach selects the binary variables to be re-optimized in an MIP model of a lot sizing problem based on the interrelatedness of binary setup variables in the constraints of the model rather than based on three problem-specific decompositions (product, resource, and process-oriented decompositions). Our FO approach is thus more general than theirs and can be applied to other 0–1 MIP models. Numerical experiments on benchmark instances show that our FO approach can obtain a better solution in a similar computation time for most instances compared with that found by the approach of Helber and Sahling. Moreover, our VNS approach can further improve the solution obtained by the FO by diversifying the search space. It outperforms the variable neighbourhood decomposition search approach proposed by Zhao et al. [5] for the MLCLSP. For the MLCLSP-L, numerical experiments on benchmark instances show that our FO approach is competitive with the fix-and-optimize approach of Sahling et al. [4] and the corridor approach of Caserta and Voß [6].

The rest of this paper is organized in seven sections. Section 2 presents the literature related to the present work. Section 3 formulates the MLCLSP and the MLCLSP-L. Our FO and VNS approaches are described in Sections 4 and 5, respectively. Sections 6 and 7 present numerical experiments of the two approaches on benchmark instances of the MLCLSP and the MLCLSP-L. The concluding section provides some remarks on future research.

2. Related literature

The MLCLSP was first introduced by Billington et al. [7], who proposed a 0–1 mixed integer programming model for the problem. Since then, various models and methods have been proposed to solve the problem. Akartunali and Miller [8] presented an extensive survey of these models and established relationships between them. Buschkühl et al. [9] gave a comprehensive review of various solution methods for the MLCLSP, whereas the review of Jans and Degraeve [10] focused on meta-heuristics for the MLCLSP and other lot sizing problems. As for the MLCLSP-L, its studies were more recent with less literature. A review of the studies before 2008 was given by Tempelmeier and Buschkühl [11]. Models and algorithms for capacitated single level lot sizing problems were reviewed by Karimi et al. [12], and those for coordinated deterministic dynamic demand lot-sizing problems were reviewed by Robinson et al. [13]. In the latter problems, a joint shared fixed setup cost is incurred each time one or more items of a product family are replenished, and a minor setup cost is charged for each item replenished. In the following, we only review previous studies on the MLCLSP and the MLCLSP-L that are related to our present work, especially the studies on mixed integer programming (MIP) based heuristics and variable neighbourhood search approaches for the two problems.

MIP-based heuristics for the MLCLSP or the MLCLSP-L solve a series of MIP subproblems (submodels) whose number of binary variables is much less than that of the original MIP model of the problem. Among these heuristics, relax-and-fix approaches and fix-and-optimize approaches are mostly used. Relax-and-fix heuristics reduce the number of binary variables to be optimized simultaneously in an MIP model of the MLCLSP by dividing it into several subproblems based on a time-oriented decomposition with moving time windows (Belvaux and Wolsey [14], Stadtler [15], Sürie and Stadtler [16]).

Pochet and Wolsey [17] may be the first ones who used an iterative fix-and-optimize approach to solve a production planning problem. This approach iteratively solves a series of MIP subproblems which optimize only a small subset of binary setup variables of the original MIP model of the problem. The MIP subproblem in each iteration is derived by fixing most binary variables of the original MIP model to their values obtained in the last iteration. This approach is easy to be implemented with a commercial MIP solver such as CPLEX or XPRESS. Later, Helber and Sahling [3] proposed a fix-and-optimize approach for the MLCLSP. Their approach considers three types of subproblems defined based on three problem-specific decompositions (i.e., product, resource, and process-oriented decompositions). Each subproblem of the first type is derived by fixing all setup variables except the setup variables related to a product. Each subproblem of the second type is derived by fixing all setup variables except the setup variables related to a resource and four periods. Each subproblem of the third type is derived by fixing all setup variables except the setup variables related to a product, one of its immediate successors, and one half of the planning horizon. Numerical experiments show that the fix-and-optimize approach outperforms the Lagrangean heuristic of Tempelmeier and Derstroff [1] and the fix-and-relax heuristic of Stadtler [15]. Sahling et al. [4] applied a similar fix-and-optimize approach to the MLCLSP-L and obtained very good results on the 1920 benchmark instances proposed by Tempelmeier and Buschkühl [11]. Stadtler and Sahling [18] presented a hybrid approach based on fix-and-relax and fix-and-optimize for a lot-sizing and scheduling problem of multi-stage flow lines with zero lead times. In addition, Chen and Chu [19] proposed a hybrid approach that combines Lagrangian relaxation with a linear programming pivot-based local search for the MLCLSP without setup time. Caserta and Voß [6] proposed an MIP-based corridor approach for the MLCLSP-L based on soft variable fixing.

Various meta-heuristics have also been used to solve the MLCLSP or other lot sizing problems (Buschkühl et al. [9] and Jans and Degraeve [10]), one of them is the variable neighbourhood