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A note on an economic lot size model for price-dependent demand under quantity and freight discounts

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ABSTRACT

In this study, we revisit research contributed by Burwell et al. (1997. International Journal of Production Economics 48, 141–155), where an economic lot size model for price-dependent demand under quantity and freight discounts was proposed. Specifically, for the cases of mixed discounts in which the quantity discounts are either of the incremental or all-units variety and the freight discounts are of the opposite type, we first provide counterexamples to show that adopting the existing algorithm to determine overall optimal lot size and selling price may not achieve the goal of maximizing profit. We then propose an easy to understand computational algorithm to obtain an exact solution.

1. Introduction

Discounts often exist in many business activities, including purchasing and transportation, which are recognized as important factors for inventory control. In order to generalize the applications of inventory models, several researchers have incorporated quantity and freight discounts simultaneously into various inventory systems. As mentioned by Toptal (2009), all-units discounts and incremental discounts are the two discounting schemes that are commonly seen in the industry and investigated in the literature, and in fact these structures are not only used for wholesale pricing by vendors, but they are also adopted by common carriers. Tersine and Barman (1991) initially conducted such a study on the classical EOQ system with constant demand, where both the unit procurement cost and the unit shipping cost received discounts via either the incremental or all-units discount schedule, and hence four possible discount configurations were considered. Darwish (2008) did a similar study based on the stochastic demand (Q, r) inventory system. Burwell et al. (1997) addressed the same issues but considered the price-dependent demand environment. Although previous studies have developed the solution procedures and algorithms to determine optimal solutions for all possible discount configurations, for the numerical experiments, only Burwell et al. (1997) applied the algorithm to solve each discount case, while others were focused on the dual all-units discount case. Additionally, in the literature many related studies (e.g., Swenseth and Godfrey, 2002; Abad and Aggarwal, 2005; Kim and Hwang, 2008; Toptal, 2012; Hua et al., 2012) have noted the research of Burwell et al. (1997), but none have examined the appropriateness of their algorithms. In this study, we explore this issue and develop an improved algorithm for mixed discount cases.

2. A brief review of Burwell et al.

For the sake of clarity and to make the analysis tractable, we first briefly review the work of Burwell et al. (1997), where they considered a lot size inventory model in which price determines demand and the supplier offers either incremental or all-units quantity and freight discounts.

For four discount configurations, Burwell et al. began by addressing the case of dual incremental discounts. Under a restructured discount schedule, for a fixed combined unit cost ($s=s_i$), the problem is to find optimal lot size (Q) and selling price (p) such that the profit $\Pi(Q,p)$ earned by the retailer is maximized.

$$\Pi(Q,p)=\left\{p-s-vrt-C_m-\frac{[H(1+rt)+Z+C_o]}{Q}\right\}D(p)-(H+Z+sQ)\frac{R}{2} \tag{1}$$

where $H=\Sigma_{b=1}^{i}(q_{s_b}-1)(v_{b-1}-v_b)$ and $Z=\Sigma_{b=1}^{i}(q_{s_b}-1)(g_{b-1}-g_b)$.

Note that the notations used here (list in Appendix A) are exactly the same as Burwell et al. and for the case of all-units quantity (or freight) discounts, $H=0$ (or $Z=0$).

Using a method similar to Abad (1988a, 1988b), Burwell et al. analyzed the properties of profit function (1) and then developed an iterative procedure to calculate Q^* and p^* for a given combined cost $s=s_i$, according to the results of first-order conditions:

$$p+\frac{D(p)}{D'(p)}=s+vrt+C_m+\frac{[H(1+rt)+Z+C_o]}{Q} \tag{2}$$

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and

$$Q=\sqrt{\frac{2D(p)[H(1+rt)+Z+C_o]}{sR}}. \quad (3)$$

We note that the above two equations are rewritten from (4) and (5) given in Burwell et al. (1997) (p. 144, where the term D/Q in (5) should be corrected as D/Q^2). Obviously, when $s=s_i$, Q^* is valid if $q_{s_i}\le Q^*<q_{s_{i+1}}$, and all the valid Q^* (corresponding to possible combined costs) are candidates for overall optimum.

The next task is to find other possible candidates, if they exist. For the case of dual incremental discounts, by the fact that the overall profit function is continuous over all positive values for Q, the break-points cannot be candidates for an overall optimum, thus the overall optimum is determined by comparing the valid $\Pi(Q^*,p^*)$. For the case of dual all-units discounts ($H=0$ and $Z=0$), the overall profit function is discontinuous at break-points; after the largest valid Q^* is found, the break-points larger than it are selected as candidates. For each case, Burwell et al. (1997) mentioned that the same condition exists in the constant demand case, and hence, except the procedure of determining (Q^*,p^*) for the given s, an algorithm similar to Tersine and Barman (1991) was developed.

For a case of mixed discounts ($H=0$ or $Z=0$), the overall profit function is discontinuous at primary break-points (we call it 'PBP' in short) in the combined discount schedule associated with the all-units discount schedule, and is continuous between PBPs. Again, Burwell et al. referred to Tersine and Barman to develop an algorithm with greater details (see Appendix B, where G denotes the set of indices I in the combined discount schedule associated with PBPs); specifically, similar to the dual all-units discount case, the PBP larger than a valid Q^* having the largest profit $\Pi(Q^*,p^*)$ was selected as a candidate.

We remark that for the cases of dual discounts, the above criterions of determining which or whether the break-points are candidates have widely been adopted in the classical EOQ model with quantity discounts (e.g., Hadley and Whitin, 1963). Also, since the property of combined discount structure (purchase and shipping) is identical to that of single discount structure (purchase), following the analytical works of Abad (1988a, 1988b), it can be shown that Burwell et al.'s approach is valid. But, for the cases of mixed discounts, there seems to be no analytical work to support them; the criterion to select possible candidates is questionable. In the next section, we are able to show this finding numerically. For more analysis about Burwell et al.'s approach, whether valid or not, see Appendix C.

Table 1
All-units quantity and incremental freight-weight discount schedules.

Quantity discount schedule			Freight-weight discount schedule		
j	Q	v	j	W	Y
0	$0<Q<1000$	\$4.5	0	$0<W<1000$	\$0.50
1	$1000\le Q<10{,}000$	\$4.0	1	$1000\le W<2500$	\$0.40
2	$10{,}000\le Q$	\$3.7	2	$2500\le W$	\$0.30

3. Counterexamples

For the cases of mixed discounts, this section provides two counterexamples to illustrate that the goal of maximizing profit cannot always be achieved by Burwell et al.'s (1997) algorithm.

Example 1. Assume that the supplier offers an all-units quantity discount schedule ($H=0$) and an incremental freight-weight discount schedule. We take the data, including the discount schedules (list in Table 1), the demand function $D=\alpha p^{-\beta}$ with $\beta=3$, the problem parameters $R=0.4$, $r=0.2$, $t=1/52$, and $C_m=\$1$, and the product's weight of 2 lb/unit, from Burwell et al., but consider a scenario for which $C_o=\$250$ (instead of \$150) and $\alpha=2{,}000{,}000$ (instead of 3,375,000). The combined discount schedule and the computing results from Burwell et al.'s algorithm are listed in Table 2.

First, we follow Burwell et al.'s illustration to determine an overall optimal solution. The set G is composed of indices {0, 2, 4} with respective minimum order levels {0, 1000, 10000}. From Table 2, the feasible Q with the largest profit ($Q=1314$ and $\Pi=\$6758$) is found among the indices associated with the PBP index, 2. Hence, the possible optimal values for Q are 1314 and 10,000. The highest profit among these two lot sizes is \$6758. Therefore, the optimal solution is to let $Q=1314$ and $p=\$9.11$.

Next, let us consider PBP=1000, which is smaller than $Q=1314$. For $Q=1000$, by (2) and (1) we obtain $p=\$9.25$ and $\Pi=\$6815$ ($>\$6758$). Thus, the exact overall optimal solution should be $Q=1000$ and $p=\$9.25$. To gain more insight of solutions, for various values of lot sizes, we calculate the corresponding prices and profits. The results are displayed in Figs. 1 and 2. From Fig. 2, it is clear that the maximum profit occurs at $Q=1000$ rather than $Q=1314$.

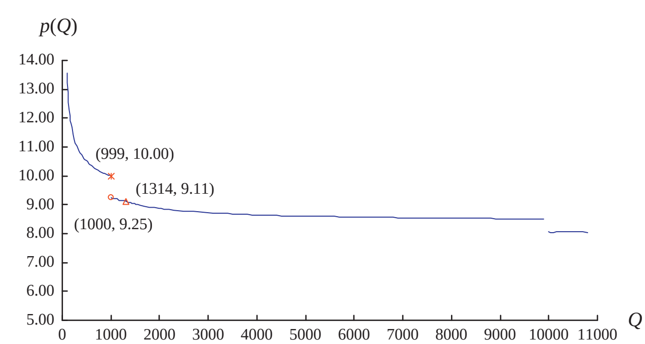

Fig. 1. Lot size Q vs. price $p(Q)$.

Table 2
Combined cost discount schedule and computing results.

Combined cost discount schedule with fixed extra shipping cost Z							Computing results			
I	Q	s	v	g	H	Z	Q	p	Valid	Π
0	$0<Q<500$	\$5.5	\$4.5	\$1.0	\$0.0	\$0.0	639	\$10.36	–	–
1	$500\le Q<1000$	\$5.3	\$4.5	\$0.8	\$0.0	\$99.8	796	\$10.13	Yes	\$5626
2	$1000\le Q<1250$	\$4.8	\$4.0	\$0.8	\$0.0	\$99.8	956	\$9.27	–	–
3	$1250\le Q<10{,}000$	\$4.6	\$4.0	\$0.6	\$0.0	\$349.6	1314	\$9.11	Yes	\$6758
4	$10{,}000\le Q$	\$4.3	\$3.7	\$0.6	\$0.0	\$349.6	10,000	\$8.06	–	\$1589