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A hierarchical copula-based world-wide valuation of sovereign risk

Enrico Bernardi, Federico Falangi, Silvia Romagnoli *

University of Bologna, Department of Statistics, via Belle Arti 41, 40126 Bologna, Italy

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ABSTRACT

We propose a new model for the aggregation of risks that is very flexible and useful in high dimensional problems. We propose a copula-based model that is both hierarchical and hybrid (HYC for short), because: (i) the dependence structure is modeled as a hierarchical copula, (ii) it unifies the idea of the clusterized homogeneous copula-based approach (CHC for short) and its limiting version (LHC for short) proposed in Bernardi and Romagnoli (2012, 2013). Based on this, we compute the loss function of a world-wide sovereign debt portfolio which accounts for a systemic dependence of all countries, in line with a global valuation of financial risks. Our approach enables us to take into account the non-exchangeable behavior of a sovereign debts' portfolio clustered into several classes with homogeneous risk and to recover a possible risks' hierarchy. A comparison between the HYC loss surface and those computed through a pure limiting approach, which is commonly used in high dimensional problems, is presented and the impact of the concentration and the granularity errors is appreciated. Finally the impact of an enlargement of the dependence structure is discussed, in the contest of a geographical area sub-portfolios analysis now relevant to determine the risk contributions of subgroups under the presence of a wider dependence structure. This argument is presented in relation to the evaluation of the insurance premium and the collateral related to the designed project of an euro-insurance-bond.

1. Introduction

A comprehensive evaluation of the risks that governments could default on their debts became crucial during the recent market global economic decline that began in December 2007. The recession, beginning in the United States in December 2007 as a consequence of the bursting of the US housing bubble, became international in September 2008. Moreover the failure of three of the major US investment banks, increased the instability of the global financial system characterized by an exceptionally high level of both private and public debt in the US and in many other countries. This excessive debt played an important role in causing bank crises, that progressed to sovereign debt problems, and appeared first in the euro-area. Many European countries embarked on austerity programs which contributed to improve their budget deficits from 2010 to 2011 relative to GDP. Despite this effort, the eurozone unemployment reached record levels during the 2012, indicating that GDP growth was not sufficient to support the variation in the debt-to-GDP ratio. On the other hand financial crisis did not affect emerging and developing countries, like the Africa area, that are not strictly integrated in the world market. The global answer to the crisis focused on the lack of transparency about banks' risk exposures that disable the markets from correctly pricing risks. European regulators introduced Basel III whose novelty did not address the problem of faulty risk-weighing, leading indeed to excessive lending to risky governments (see Nouy, 2012 for a detailed discussion about the regulations required from capital institutions to hold adequate regulatory capital associated with sovereign exposures).

The complex interactions between the economic and financial variables remind us how important it is to improve the modeling of financial crises and sovereign risk. Gray et al. (2007) proposed a model based on the theory and practice of modern contingent claim analysis, where the sectors of a national economy and the sovereign balance sheet are considered as dependent portfolios of assets, liabilities and guarantees. A systemic version of this model appeared in Gray and Jobst (2011) where a multivariate approach represents the interactions and the feedback between the financial sector and the sovereign.

We propose a new model (HYC for short), that is essentially *copula-based*. Multivariate copulas (see Nelsen, 2006 and Joe, 1997), allowing for a very flexible dependence structures and facilitating statistical parameters' estimations, are commonly used in quantitative finance (see Cherubini et al., 2004 for standard

* Corresponding author.
E-mail address: silvia.romagnoli@unibo.it (S. Romagnoli).

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applications of copula in finance and Cherubini et al., 2012 for the new frontiers of the dynamic approach to copulas). Multivariate copulas has recently become the most significant new statistical tool to represent the co-movement between markets, risk factors and other relevant variables studied in finance. The questions concerned about correlations among input variables have become important in quantitative risk assessment; a copula-based Monte Carlo method is proposed in Haas (1999) and an interesting discussion about the impact of the choice of the marginal distributions versus the choice of the dependence structure on the resulting distribution of risk can be found in Bukowski et al. (1995). A very flexible class of mixed C-vines copulas permitting the variables to be ordered according to their influence, was proposed in Czado et al. (2012) in an extensive simulation study involving US exchange rates. Along the same multivariate approach, we propose here a new model to compute a world-wide loss distribution which accounts for a systemic dependence of all the countries in line with a global valuation of financial risk.

Moreover a second peculiar aspect of the suggested model is to be a *hierarchical* approach, in line with Savu and Trede (2010), since it considers more than one level of aggregation, in such a way as to represent the effect of several risk factors acting differently on a set of sovereign risk's classes. A common way to define risk's classes is to implement clustering methods: these have been extensively proposed in literature, essentially to diminish the complexity of the problem and to identify common factors into large data sets. Following the clustering approach of Bernardi and Romagnoli (2011), on the construction of a clustered copula function, related to the choice of the metric and the number of clusters followed by the homogenization of groups, we focus here on recovering a world-wide sovereign loss distribution. In order to do this we propose a *hybrid* version of the CHC and the LHC models, appeared in Bernardi and Romagnoli (2012, 2013). The model is hybrid, because at the lowest level of risk and depending on the cardinality of the risk's class, it selects when to compute the loss pdf through a pure limiting model (see Vasicek, 1987, 1997 and Schönbucher, 2004), PL in the following, or rather through a clustered one, where the granularity of groups is taken into account.

Our field is strictly linked with the literature on concentration risk, decomposed into the granularity and the sector concentration risks respectively, introduced here in the proposed methodology. Therefore granularity is not taken into account in large groups, while it is considered only into the small ones; anyway a semi-granularity adjustment (see Gordy, 2000, 2003, 2004) may be also introduced into the *within large groups* setting. On the other hand, here we are interested also in the sector concentration risk, in the same spirit of multi-factor models (see Pykhtin, 2004 and Tasche, 2006), that can be evaluated and recovered as marginal component of our model.

We therefore propose an empirical study based on a clustered data set of sovereign debt products, and we discuss the impact of the sector concentration and granularity risk into the calculation of its loss distribution, by a comparison between the new HYC aggregation methodology and the most commonly used PL approaches. Furthermore, a second contribution is related to a geographical area sub-portfolios analysis, aimed at stressing the impact of an enlargement of the dependence structure in determining the risk contributions and the insurance premiums related to a euro-insurance-bond, a new project of debt instrument that could represents a possible solution to sovereign debt crisis, providing an insurance against the eurozone bankruptcy. It is then evident how the link among economies and the systemic and the contagion effects among them, are relevant factors in order to evaluate sovereign risks and their risk's contributions based on several and progressively wider dependence structures.

The paper is organized as follows: in Section 2 we present and discuss the HYC approach concerning the within and the between groups' dependence structure to recover the loss distribution. Moreover we provide a comparison of the HYC model to other aggregating methodologies appeared in literature or used by regulator in the contest of a proper insurance application. In Section 3 we propose a world-wide sovereign debt large portfolio empirical application; a comparative analysis with respect to the PL and LHC Gaussian and Archimedean approaches and a geographic areas' sub-portfolio analysis is outlined. Moreover the impact of an enlargement of the dependence structure is evaluated in defining the elements, designing a euro-insurance-bond, i.e. the insurance premium and the risk contribution. Section 4 concludes.

2. A copula based hierarchical approach

The proposed HYC model is a very flexible copula-based model particularly apt to deal with high dimensional problems, since it works by aggregating classes of risk defined through a clusterization. As we will see in the following empirical analysis, the best kind of copula that is usually chosen, corresponds to the one best fitting our data set. The flexibility of this approach allows us to aggregate coherently within a hierarchy of risks's clusters; the constraints to be respected in the construction of a hierarchical copula are the necessary ones to abide by the compatible rules, clearly explained e.g. in Hofert (2010). Therefore the basic idea is to introduce a hierarchical risks' structure allowing us to take into account the non-exchangeability[1] of the variables and the grains[2] of such classes. It is important to highlight that we take into account the sector concentration, but we consider the right granularity only for groups, whose cardinality is smaller than a fixed threshold[3] k, and that are deemed *small*, while for greater cardinalities we use a limiting approach to represent the dependence structure within the groups.

On the other hand a second point that distinguish HYC approach from other copula-based ones concerns its distinguished feature, based on a combinatoric algorithm capable to picture all possible scenarios concerning a random event affecting a set of variables, whose dependence structure is known. Thanks to this peculiar feature, HYC model is not affected by any Monte Carlo or simulation errors. In this view HYC model can be considered both copula-based and scenario-based.

The HYC model is built up in three steps: a clustering phase, a *within class* computing step and an *between classes* aggregation step.

2.1. Clustering methods to recover homogeneous classes of risk

The implementation of the HYC methodology is concerned basically with the reduction of the complexity of the problem with respect to the dimension of the data set. This goal may be reached by a clustering and a homogenization procedure.

Given a set of data, we assume to partition it into homogeneous clusters, such that patterns within a cluster are more similar

[1] The concept of non-exchangeability corresponds to having a hierarchy of risks that has to be respected in the aggregation process, since no exchange is permitted into the hierarchy's positions. These kinds of copulas are classified as fully or partially nested copulas; partially nested ones preserve the exchangeability among groups because they are linked at systemic level by the same kind of dependency.

[2] The granularity of a risk's class is introduced if every component of the class is considered in its individuality. Obviously the fine grain of the class is lost when we model it through a limiting approach, that is recovered and proved for a cardinality that goes to $+\infty$.

[3] In the following empirical analysis we set $k = 20$ in line with the argument of Schönbucher (2004).